Honors Calculus Semester 1 Exam Review: 30 questions, multiple choice, calculators allowed
TEXTBOOK SOLUTIONS WITH STEPS: www.calcchat.com/book/Calculus-7e/

Limits and Continuity 45%

- Given a graph of a function, be able to determine its limits (one-sided and two-sided).
  - p. 55: #9-18 See what y-value the graph has around the indicated x-value.
- Determine the limits of a piecewise function (including absolute value functions)
  - $f(x) = \frac{3x+9}{|x+3|}$; $\lim_{x \to -3^+} f(x)$; (answer: 3) $\begin{cases} \frac{3(\cancel{x+3})}{\cancel{x+3}}, & x \geq -3 \\ \frac{3(\cancel{x+3})}{-(\cancel{x+3})}, & x < -3 \end{cases} \Rightarrow \begin{cases} 3 & x \geq -3 \\ -3 & x < -3 \end{cases} \leftarrow \lim_{x \to -3^+}$
  - $g(x) = \begin{cases} x^2 + 2, x \neq 0 \\ 0, x = 0 \end{cases}$; $\lim_{x \to 0} g(x)$; (answer: 2) "crumbs" around 0 ≠ exactly 0.

Incomprehensible math cloud of misery ↓ $\lim_{x \to c}$ Stuck? Graph by putting it in Y_1 in calc. Then, $\boxed{\text{TRACE}}$ x-values around c.

- Find limits of functions analytically:
  - Using properties of limits: p. 65: 37-40 → common sense
  - By direct substitution: p. 65: 15-22 → plug it in, hopefully no division by 0.
  - By factoring: p.66: 45-52 → factor, cancel, then plug in.
  - By rationalization: p.66: 53-56 → see p. 3 for example
  - Of special trig limits: p. 66: 67-70 → $\lim_{x \to 0} \frac{\sin x}{x} = 1$.
- Determine the continuity of a function: (limit from the left = function at point = limit from right)
  - $f(x) = \begin{cases} x^2 + 2x, & \text{if } x \leq -2 \\ x^3 - 6x, & \text{if } x > -2 \end{cases}$ See pg. 3 for work.
    (*answer*: *discontinuity at* $x = -2$ *because* $0 = 0 \neq 4$)
- Find and classify discontinuities of functions: *jump* (left limit≠right limit), *removable* (limit exists, but does not equal function), *infinite* (one or both of the left/right limits approaches infinity; v. asymptote)
  - p. 77: 37-47 (except 43 & 44)
- Given a function, be able to find its limit that results in infinity (vertical asymptote)
  - $f(x) = \frac{-2}{x+5}$; $\lim_{x \to -5^+} f(x)$ (*answer*: $-\infty$) See pg 3.
- Given a function, find its limits *at* infinity. (Horizontal asymptotes)
  - Same degrees: H.A. is y = (ratio of leading coefficients)
  - Numerator degree is higher: no H.A.; or, lim = ∞
  - Denominator degree is higher: H.A. is y = 0
  - p. 199: 19-26
- Find the value of *c* that will make a function continuous:
  - $p(x) = \begin{cases} 3x^2 - 2, if\ x \leq 4 \\ 4cx - 2, if\ x > 4 \end{cases}$ (answer: c = 3) See pg. 4.

Differentiation: Derivatives, Tangent Lines, and Differentiability: 55%

- Use derivative rules to find derivatives of functions:

Function	Derivative
F(x)	Limit definition: F'(X) = $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
F(x) = c	0
F(x) = cx	c
F(x) = cx^n	cnx^{n-1}
F(x) = $(G(x))^n$	$n \cdot (G(x))^{n-1} \cdot G'(x)$
H(x) = F(x) + G(x)	$F'(x) + G'(x)$
H(x) = F(x)*G(x)	$F'G + FG'$
H(x) = F(x)/G(x)	$\frac{F'G - FG'}{G^2}$

H(x) = F(G(x))	$F'(G(x)) \cdot G'(x)$
F(x) = sin(x); cos(x); tan(x)	$\cos(x)$; $-\sin(x)$; $\sec^2(x)$
F(x) = csc(x); sec(x); cot(x)	$-\csc(x)\cdot\cot(x)$; $\sec(x)\tan(x)$; $-\csc^2(x)$
H(x) = e^(F(x))	$e^{f(x)} \cdot f'(x)$
H(x) = ln(F(x))	

$\frac{1}{f(x)} \cdot f'(x)$

- Practice with above rules: p. 113: 39-52
- Use the limit definition of derivative to find the derivative of a function.
  - $\lim_{h \to 0} \frac{5(x+h)^2 - 3(x+h) + 7 - 5x^2 + 3x - 7}{h}$ (answer: $10x - 3$) power rule
    $-(5x^2-3x+7) \rightarrow$ this is $f(x)$
- Given a table of values, find the indicated derivative (using the power rule, product rule, quotient rule, and or chain rule) (answers ↓)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	2	2
2	2	-1	4	0
3	1	$\frac{1}{2}$	2	$-\frac{3}{2}$
4	3	2	1	-1

Part 1) Given $h_1(x) = f(x) \cdot g(x)$, find $h_1'(2)$

Part 2) Given $h_2(x) = \frac{f(x)}{g(x)}$, find $h_2'(4)$

Part 3) Given $h_3(x) = (f(x))^2$, find $h_3'(1)$

Part 4) Given $h_4(x) = f(g(x))$, find $h_4'(3)$

See pg 4.

$h_1'(2) = -4$

$h_2'(4) = 5$

$h_3'(1) = -6$

$h_4'(3) = \frac{3}{2}$

  - Procedure: Use appropriate rule to find h'(x) in terms of f, g, f', and g'. Plug in the given x value into your functions/derivatives. Use the table to find what these are.
- Use the product and quotient rules to find derivatives (possibly involving trigonometric functions)
  - Procedure: Specify F and G. Find F' and G'. Plug in these 4 bits into the appropriate formula. Simplify.
  - p. 124: 1-12; 39-45

See pg 5

- Use the chain rule to find derivatives (possibly involving trigonometric functions)
  - Procedure: Derivative of outside function (keeping inside as is), times the derivative of inside.
  - p. 133: 7-16
- Use implicit differentiation to find dy/dx of a given function. Then, find the slope at a specified point.

See pg 6 for ex.

  - Procedure: Use derivative as operator to both sides of function. Every term with x as only variable are done as usual.
  - Terms with a y in it require the chain rule, meaning dy/dx is multiplied to what the derivative of the y term gives you.
  - Use algebra to isolate and solve for dy/dx. Then plug in the (x,y) value of the point into this, and this is the slope of the curve.
  - P. 142: 21-28
- Find the equation of the tangent line of a function at a specified x-value.

See p. 7

  - Procedure: Plug in given x into given function, find y. This is your (x_1, y_1).
  - Find the derivative of the given function. Plug in the given x into this derivative. This value is your *m*.
  - Plug in (x_1, y_1) and *m* into $y - y_1 = m(x - x_1)$
  - P. 114: 53-56. (Note: these problems give you the y-value. Pretend you don't have it when working on them).

See pg. 7

- Find the slope of the line tangent to the function at a specified x-value.
  - Procedure: Very easy. Just take the derivative, and plug the given x-value into it. This is your answer.

Rationalization

$$\lim_{x\to 4} \frac{x-4}{(\sqrt{x+5}-3)} \cdot \frac{\sqrt{x+5}+3}{(\sqrt{x+5}+3)} \Rightarrow \frac{(x-4)(\sqrt{x+5}+3)}{x+5-9} = \frac{\cancel{(x-4)}(\sqrt{x+5}+3)}{\cancel{x-4}}$$

Foil where the sq·rt is. Don't foil the other part.

$$\lim_{x\to 4} (\sqrt{x+5}+3)$$

plug in

$$\sqrt{4+5}+3$$
$$\sqrt{9}+3$$
$$3+3 = 6$$

Continuity

$$f = \begin{cases} x^2+2x, & x \le -2 \\ x^3-6x, & x > -2 \end{cases}$$

Easy method? Plug -2 into both equations. this is to see if $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$

$\boxed{-2^-}$ $\boxed{-2^+}$

$(-2)^2 + 2(-2)$ $(-2)^3 - 6(-2)$

$4-4$ $-8+12$

$0 \neq 4$, Jump Discontinuity..

Infinity?

$$\lim_{x\to -5^+} \frac{-2}{(x+5)^2}$$

- Plug in? Divide by 0 !!
- Can't factor...
- Plug in $-5^+ \approx -4.999$

$$\frac{-2}{(-4.999+5)^2} = \frac{-2}{(0.001)^2} = \frac{-2}{0.0001} = -\infty$$

$\frac{\text{neg}}{\text{pos}}$

Make a function continuous.

$$p(x)=\begin{cases}3x^2-2, & x\le 4\\ 4cx-2, & x>4\end{cases}$$

Easy Method? Plug in 4 in both functions. Only plug it in for x. "c" stays put. Set equal and solve.

$$3(4)^2-2 = 4c\cdot 4-2$$
$$3\cdot 16-2 = 16c-2$$
$$48-2 = 16c-2$$
$$46 = 16c-2$$
$$+2 \qquad +2$$
$$\frac{48}{16}=\frac{16c}{16} \Rightarrow \boxed{c=3}$$

$\boxed{\text{Tables}}$

① $h=f\cdot g$ (product rule)

$h'=f'g+fg'$

$h'(2)=f'(2)\cdot g(2)+f(2)\cdot g'(2)$

use table to find all 4 things.

$-1\cdot 4+2\cdot 0$

$-4+0=-4$

② $h(x)=\dfrac{f(x)}{g(x)}$ Quot. rule.

$$h'(x)=\frac{f'(x)g(x)-f(x)\cdot g'(x)}{(g(x))^2}$$

$$h'(4)=\frac{f'(4)g(4)-f(4)\cdot g'(4)}{(g(4))^2}$$

use table.

$$\frac{2\cdot 1-3\cdot -1}{1^2}$$ (order of operations)

$$=\frac{2--3}{1}$$ $2+3=5$

③ $h(x)=(f(x))^2$ power rule/chain rule

$$h'(x)=2(f(x))^1\cdot f'(x)$$

$$h'(1)=2(f(1))\cdot f'(1)$$

(use table)

$2\cdot 3\cdot -1 = -6$

④ $h(x)=f(g(x))$ "party"

$$h'(x)=f'(g(x))\cdot g'(x)$$

$$h'(3)=f'(g(3))\cdot g'(3)$$ TABLE

$$h'(3)=f'(2)\cdot -\frac{3}{2}$$ table

$$-1\cdot -\frac{3}{2}=\frac{3}{2}$$

these are Chain rule problems.

CHAIN RULE

$$y = \sqrt{\cos(2x)}$$

Rewrite

$$y = \left(\cos\left(2x\right)\right)^{1/2}$$

after-party

party on inside

outside

$$\frac{dy}{dx} = \frac{1}{2}\left(\cos(2x)\right)^{-1/2} \cdot -\sin(2x) \cdot 2$$

chain

chain

$$\frac{1}{2\left(\cos(2x)\right)^{1/2}} \cdot -2\sin(2x)$$

$$\frac{-\cancel{2}\sin(2x)}{\cancel{2}\sqrt{\cos(2x)}} = \boxed{\frac{-\sin(2x)}{\sqrt{\cos(2x)}}}$$

Implicit Diff.

Find $\frac{dy}{dx}$.

$$x^3 - y^3 = \boxed{3x}\boxed{y}$$

$f \quad g$

$f' = 3$

$g' = 1 \cdot \frac{dy}{dx} = \frac{dy}{dx}$

product rule: $f'g + fg'$

$$3x^2 - 3y^2 \cdot \frac{dy}{dx} = 3 \cdot y + 3x\frac{dy}{dx}$$

move $\Rightarrow$ keep keep move $\Leftarrow$

} use algebra to isolate "$\frac{dy}{dx}$"

$$-3y^2\frac{dy}{dx} - 3x\frac{dy}{dx} = 3y - 3x^2$$

factor out dy/dx.

$$\frac{\frac{dy}{dx}\cancel{(-3y^2-3x)}}{\cancel{(-3y^2-3x)}} = \frac{3y - 3x^2}{-3y^2-3x}$$

$$\frac{dy}{dx} = \frac{-\cancel{3}(x^2-y)}{-\cancel{3}(x+y^2)}$$

Clever factoring

$$\boxed{\frac{dy}{dx} = \frac{x^2-y}{x+y^2}}$$

Tangent line

Find the equation of the line tangent to $y = \sin(2x)$ at $x = \pi/4$.

$$\ast\ y - y_1 = m(x - x_1)$$

Point $y = \sin(2 \cdot \pi/4) = \sin\left(\frac{\pi}{2}\right) = 1$

$\boxed{(\pi/4, 1)}$ (x_1, y_1)

Slope

$$\frac{dy}{dx} = 2\cos(2x)$$

plug in $x = \pi/4$

$$\frac{dy}{dx} = 2\cos(2 \cdot \pi/4)$$
$$= 2\cos(\pi/2)$$
$$= 0 \quad m$$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 0\left(x - \frac{\pi}{4}\right)$$
$$y - 1 = 0$$
$$\boxed{y = 1}$$

Slope

① find $f'(4)$ if $f(x) = x^2 + 3x - 1$

$$f'(x) = 2x + 3$$
$$f'(4) = 2 \cdot 4 + 3 = 8 + 3 = 11$$

② Find $f'(5)$ if $f(x) = -3x - 7$

$$f'(x) = -3$$
$$f'(5) = -3 \ldots \text{So} \ldots -3$$

makes sense!! Slope of f is always -3.