...rcises 1–4, assume that x and y are both differentiable ...ons of t and find the required values of dy/dt and dx/dt.

...uation	Find	Given
$\sqrt{x}$	(a) $\frac{dy}{dt}$ when $x = 4$	$\frac{dx}{dt} = 3$
	(b) $\frac{dx}{dt}$ when $x = 25$	$\frac{dy}{dt} = 2$
$2(x^2 - 3x)$	(a) $\frac{dy}{dt}$ when $x = 3$	$\frac{dx}{dt} = 2$
	(b) $\frac{dx}{dt}$ when $x = 1$	$\frac{dy}{dt} = 5$
4	(a) $\frac{dy}{dt}$ when $x = 8$	$\frac{dx}{dt} = 10$
	(b) $\frac{dx}{dt}$ when $x = 1$	$\frac{dy}{dt} = -6$
$y^2 = 25$	(a) $\frac{dy}{dt}$ when $x = 3, y = 4$	$\frac{dx}{dt} = 8$
	(b) $\frac{dx}{dt}$ when $x = 4, y = 3$	$\frac{dy}{dt} = -2$

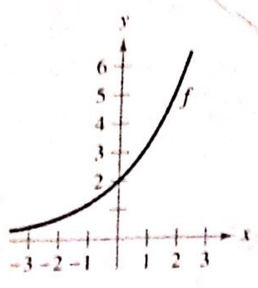
...ses 5–8, a point is moving along the graph of the func- ... that dx/dt is 2 centimeters per second. Find dy/dt for ...ied values of x.

...on	Values of x		
$+ 1$	(a) $x = -1$	(b) $x = 0$	(c) $x = 1$
$\frac{1}{+ x^2}$	(a) $x = -2$	(b) $x = 0$	(c) $x = 2$
x	(a) $x = -\frac{\pi}{3}$	(b) $x = -\frac{\pi}{4}$	(c) $x = 0$
x	(a) $x = \frac{\pi}{6}$	(b) $x = \frac{\pi}{4}$	(c) $x = \frac{\pi}{3}$

...g at the Concept

...ises 9 and 10, using the graph of f, (a) determine ... dy/dt is positive or negative given that dx/dt is ... and (b) determine whether dx/dt is positive or ...given that dy/dt is positive.

10.



...der the linear function $y = ax + b$. If x changes at a ...nt rate, does y change at a constant rate? If so, does it ... at the same rate as x? Explain.

12. In your own words, state the guidelines for solving related rate problems.

13. Find the rate of change of the distance between the origin and a moving point on the graph of $y = x^2 + 1$ if $dx/dt = 2$ centimeters per second.

14. Find the rate of change of the distance between the origin and a moving point on the graph of $y = \sin x$ if $dx/dt = 2$ centimeters per second.

15. ***Area*** The radius r of a circle is increasing at a rate of 3 centimeters per minute. Find the rate of change of the area when (a) $r = 6$ centimeters and (b) $r = 24$ centimeters.

16. ***Area*** Let A be the area of a circle of radius r that is changing with respect to time. If dr/dt is constant, is dA/dt constant? Explain.

17. ***Area*** The included angle of the two sides of constant equal length s of an isosceles triangle is θ.

(a) Show that the area of the triangle is given by $A = \frac{1}{2}s^2 \sin \theta$.

(b) If θ is increasing at the rate of $\frac{1}{2}$ radian per minute, find the rate of change of the area when $\theta = \pi/6$ and $\theta = \pi/3$.

(c) Explain why the rate of change of the area of the triangle is not constant even though $d\theta/dt$ is constant.

18. ***Volume*** The radius r of a sphere is increasing at a rate of 2 inches per minute.

(a) Find the rate of change of the volume when $r = 6$ inches and $r = 24$ inches.

(b) Explain why the rate of change of the volume of the sphere is not constant even though dr/dt is constant.

19. ***Volume*** A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is (a) 30 centimeters and (b) 60 centimeters?

20. ***Volume*** All edges of a cube are expanding at a rate of 3 centimeters per second. How fast is the volume changing when each edge is (a) 1 centimeter and (b) 10 centimeters?

21. ***Surface Area*** The conditions are the same as in Exercise 20. Determine how fast the *surface area* is changing when each edge is (a) 1 centimeter and (b) 10 centimeters.

22. ***Volume*** The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Find the rate of change of the volume if dr/dt is 2 inches per minute and $h = 3r$ when (a) $r = 6$ inches and (b) $r = 24$ inches.

→ **23.** ***Volume*** At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

24. ***Depth*** A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.

(b) At what rate is the water level rising ...

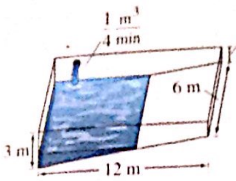


Figure for 25

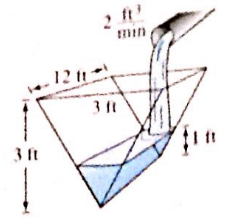


Figure for 26

26. ***Depth*** A trough is 12 feet long and 3 feet across the top (see figure). Its ends are isosceles triangles with altitudes of 3 feet.

(a) If water is being pumped into the trough at 2 cubic feet per minute, how fast is the water level rising when it is 1 foot deep?

(b) If the water is rising at a rate of $\frac{3}{8}$ inch per minute when $h = 2$, determine the rate at which water is being pumped into the trough.

27. ***Moving Ladder*** A ladder 25 feet long is leaning against the wall of a house (see figure). The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

(a) How fast is the top moving down the wall when the base of the ladder is 7 feet, 15 feet, and 24 feet from the wall?

(b) Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 7 feet from the wall.

(c) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 7 feet from the wall.

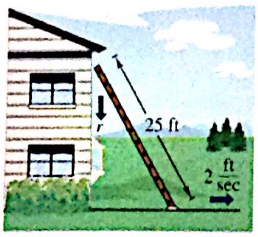


Figure for 27

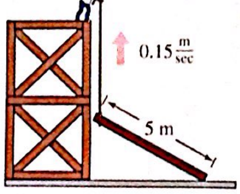


Figure for 28

FOR FURTHER INFORMATION For more information on the mathematics of moving ladders, see the article "The Falling Ladder Paradox" by Paul Scholten and Andrew Simoson in *The College Mathematics Journal*. To view this article, go to the website *www.matharticles.com*.

28. ***Construction*** A construction worker pulls a 5-meter plank up the side of a building under construction by means of a rope tied to one end of the plank (see figure). Assume the opposite end of the plank follows a path perpendicular to the wall of the

...tal change...

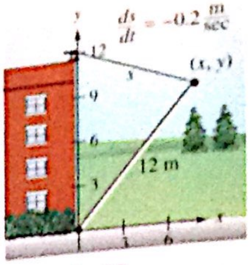


Figure for 29

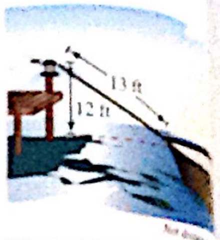


Figure for 30

30. ***Boating*** A boat is pulled into a dock by means of a w... feet above the deck of the boat (see figure).

(a) The winch pulls in rope at a rate of 4 feet per se... Determine the speed of the boat when there is 13 f... rope out. What happens to the speed of the boat as i... closer to the dock?

(b) Suppose the boat is moving at a constant rate of 4 fe... second. Determine the speed at which the winch pu... rope when there is a total of 13 feet of rope out. ... happens to the speed at which the winch pulls in rope a... boat gets closer to the dock?

31. ***Air Traffic Control*** An air traffic controller spots two p... at the same altitude converging on a point as they fly at r... angles to each other (see figure). One plane is 150 miles ... the point moving at 450 miles per hour. The other plane is ... miles from the point moving at 600 miles per hour.

(a) At what rate is the distance between the planes decrea...

(b) How much time does the air traffic controller have t... one of the planes on a different flight path?

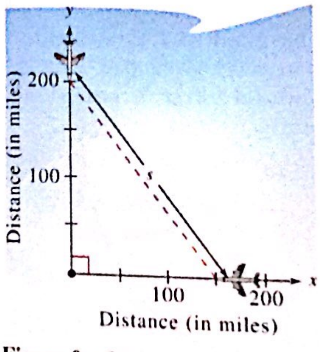


Figure for 31

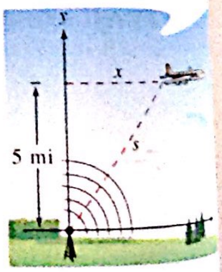


Figure for 32

32. ***Air Traffic Control*** An airplane is flying at an altitude ... miles and passes directly over a radar antenna (see fi... When the plane is 10 miles away ($s = 10$), the radar de... that the distance s is changing at a rate of 240 miles pe... What is the speed of the plane?

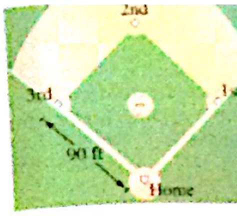


Figure for 33 and 34

34. ***Baseball*** For the ba... the player is running ... per second. Find the r... is changing when the...

35. ***Shadow Length*** A ... second away from a ... figure). When he is ...

(a) at what rate is th...

(b) at what rate is th...

36. ***Shadow Length*** ... walking at a rate of ... feet above the grou...

Figure for 36

37. ***Machine Design*** ... 1 meter have c... position of the e...

$x(t) = \frac{1}{2}\sin\frac{\pi t}{6}$

where t is the ti...

(a) Find the tim...

(b) What is the ... the y-axis?

(c) Find the s... endpoint is...

38. ***Machine Desi...*** of $x(t) = \frac{3}{5}\sin$ ...