

Excitement w/ Derivatives 5/15

556. $y = e^{2x} \quad y' = 2e^{2x}$

557. $y = e^{-\frac{3}{2}x} \quad y' = -\frac{3}{2}e^{-\frac{3}{2}x}$

558. $y = x^2 e^x$
 $f: x^2 \quad f': 2x$
 $g: e^x \quad g': e^x$

$y' = 2xe^x + x^2 e^x \rightarrow xe^x(2+x)$

559. $y = 5e^{2-x} \rightarrow y' = 5e^{2-x} \cdot (-1)$
 $y' = -5e^{2-x}$

560. $y = 8^{2x}$
 $y' = 8^{2x} \cdot \ln 8 \cdot 2$
 $2 \ln 8 \cdot 8^{2x}$
 $\ln 64 \cdot 8^{2x}$

561. $y = 3^{x^2}$
 $y' = 3^{x^2} \cdot \ln 3 \cdot 2x$

562. $y = 2^{\sin x}$
 $y' = 2^{\sin x} \cdot \ln 2 \cdot \cos x$

563. $y = 9^{-x} \rightarrow y' = 9^{-x} \cdot \ln 9 \cdot (-1)$
 $- \ln 9 \cdot 9^{-x}$
 $\ln \frac{1}{9} \cdot 9^{-x}$

564. $y = \frac{e^{5x}}{x^2}$
 $f: e^{5x} \quad f': 5e^{5x}$
 $g: x^2 \quad g': 2x$
 $y' = \frac{f'g - fg'}{g^2} = \frac{5e^{5x} \cdot x^2 - e^{5x} \cdot 2x}{x^4} \rightarrow \frac{x \cdot e^{5x} (5x - 2)}{x^3}$
 $\frac{e^{5x} (5x - 2)}{x^3}$

$$565. y = \ln x^2 = \frac{1}{x^2} \cdot 2x = \frac{2x}{x^2} = \frac{2}{x}$$

$$566. y = \ln(2-x^2) \Rightarrow y' = \frac{1}{2-x^2} \cdot -2x \Rightarrow \frac{-2x}{2-x^2}$$

$$567. y = \ln(5x+1) \Rightarrow y' = \frac{1}{5x+1} \cdot 5 \Rightarrow \frac{5}{5x+1}$$

$$568. y = \ln(\sin x) \Rightarrow y' = \frac{1}{\sin(x)} \cdot \cos(x) \Rightarrow \frac{\cos(x)}{\sin(x)} = \cot(x)$$

$$569. y = (\ln x)^2 \Rightarrow y' = 2(\ln x)' \cdot \frac{1}{x} = \frac{2}{x} \ln x$$

$$570. y = \log_9 x^{1/2}$$

$$y = \frac{1}{2} \log_9 x \rightarrow y' = \frac{1}{2x \ln 9}$$

$$570. y = \log_3(x+1) = \frac{1}{(x+1) \ln 3}$$

$$572. y = x \cdot \ln x - x \quad \begin{matrix} f': 1 \\ g': \frac{1}{x} \end{matrix}$$

$\begin{matrix} \uparrow & \uparrow \\ f & g \end{matrix}$

$$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$$

$$y' = \ln x + 1 - 1 \rightarrow \ln x$$

$$573. y = \frac{\ln x}{x^2} \quad \begin{matrix} f': \frac{1}{x} \\ g': 2x \end{matrix} \quad y' = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4}$$

$$\frac{1-2\ln x}{x^3} = \frac{x(1-2\ln x)}{x^4} = \frac{x - 2x \ln x}{x^4}$$

574. $g(x) = x^3 e^{2x}$ $f': 3x^2$
 $f \uparrow g \uparrow$ $g': 2e^{2x}$

$$\frac{3x^2 e^{2x} + x^3 \cdot 2e^{2x}}{x^2 e^{2x} (3 + 2x)}$$

575. $Z(k) = 4e^{4k^2+5}$
 $Z'(k) = 4e^{4k^2+5} \cdot 8k \rightarrow \boxed{32k \cdot e^{4k^2+5}}$

576. $f(x) = \ln(e^x + 1)$
 $f'(x) = \frac{1}{e^x + 1} \cdot e^x = \frac{e^x}{e^x + 1}$

577. $f(x) = \frac{e^x - 1}{e^x + 1} = A$ $f': e^x$
 $g': e^x$

$$\frac{e^x(e^x + 1) - (e^x - 1)e^x}{(e^x + 1)^2} \rightarrow \frac{e^x(e^x + 1 - e^x + 1)}{(e^x + 1)^2}$$

578. $k(x) = \log_3(x^2 + e^x)$
 $k'(x) = \frac{1}{(x^2 + e^x) \ln 3} \cdot (2x + e^x)$

$$k'(x) = \frac{2x + e^x}{(x^2 + e^x) \ln 3}$$

579. $R(x) = \frac{2^x - 1}{5^x}$ $f': 2^x \ln 2$
 $g': 5^x \ln 5$

$$\frac{2^x \ln 2 (5^x) - (2^x - 1) 5^x \ln 5}{(5^x)^2}$$

580. $D(x) = \ln(\ln(x))$. $\frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$

581. $A(x) = \ln(x^2 + x + 1)^2 = 2 \ln(x^2 + x + 1)$

$A'(x) = 2 \cdot \frac{1}{x^2 + x + 1} \cdot (2x + 1) \rightarrow \frac{4x + 2}{x^2 + x + 1}$

582. $g(x) = \ln(3x - 2)^{1/5}$

$g(x) = \frac{1}{5} \ln(3x - 2)$

$g'(x) = \frac{1}{5} \cdot \frac{1}{3x - 2} \cdot 3 \rightarrow \frac{3}{5(3x - 2)}$

583. $A(x) = \frac{\ln x}{x - 2}$ $f: \frac{1}{x}$
 $g: x - 2$ $g': 1$

~~$\frac{1}{x} \cdot \ln x$~~ $\frac{\frac{1}{x} \cdot (x - 2) - \ln(x) \cdot 1}{(x - 2)^2} \rightarrow \frac{(x - 2) - x \ln x}{x(x - 2)^2}$

584. $B(x) = \frac{x - 2}{\ln x}$ $f: 1$ $g: \frac{1}{x}$
 $f': 1$ $g': \frac{1}{x^2}$

$\frac{1 \cdot \ln x - (x - 2) \cdot \frac{1}{x^2}}{(\ln x)^2} \cdot x$
 $\frac{x \ln x - (x - 2)}{x(\ln x)^2}$

585. $M(x) = e^{-2x^3}$
 $M'(x) = -6x^2 e^{-2x^3}$

586. $J(x) = \frac{e^x}{x^3}$ $f: e^x$ $g: 3x^2$
 $f': e^x$ $g': 3x^2$

$\frac{e^x(x - 3)}{x^4}$ $\frac{x^2 e^x (x - 3)}{x^6}$

587. $F(x) = x^2 e^{-4 \ln x}$
 $F(x) = x^2 \cdot (e^{\ln x})^{-4}$ } prop. of exponents
 } opposites/inverses

$F(x) = x^2 (x)^{-4}$ } prop. of exp.

$F(x) = x^{-2}$

$F'(x) = -2x^{-3} =$

$\left(\frac{2}{x^3} \right)$

588. $f(x) = 10^{3x^2 - 6x}$ $f'(x) = 10^{3x^2 - 6x} \ln 10 \cdot (6x - 6)$

589. $g(x) = 3^{2x} 2^{3x^2} \rightarrow g(x) = (3^2)^x (2^3)^{x^2} \rightarrow g(x) = 9^x \cdot 8^{x^2}$

$g'(x) = f'g + fg'$

$g'(x) = 9^x \ln 9 \cdot 8^{x^2} + 9^x \cdot 8^{x^2} \cdot \ln 8 \cdot 2x$

$g'(x) = 9^x \cdot 8^{x^2} (\ln 9 + 2x \cdot \ln 8)$

$f' \uparrow g \uparrow$
 $f': 9^x \ln 9$
 $g': 8^{x^2} \ln 8 \cdot 2x$

596. $y = 3x \cdot \csc 2x$ $f'g + fg'$
 $f: 3$ $g: -\csc 2x$
 $f': 3$ $g': -\csc 2x \cdot \cot 2x \cdot 2$
 $y' = 3 \csc 2x + 3x \cdot -2 \csc 2x \cot 2x$
 $y' = 3 \csc 2x - 6x \csc 2x \cot 2x$
 $3 \csc 2x (1 - 2x \cot 2x)$

597. $y = \frac{\cot 5x}{3x^2}$ $f: -\csc^2 5x \cdot 5$
 $g: 6x$
 $-5 \csc^2(5x) \cdot 3x^2 - \cot(5x) (6x) = \frac{-15x^2 \csc^2(5x) - 6x \cot(5x)}{9x^4}$

598. $y = (\cot 5x)^{1/2} \rightarrow y' = \frac{1}{2} (\cot(5x))^{-1/2} (-\csc^2(5x)) \cdot 5$
 $-\frac{5 \csc^2(5x)}{2 \sqrt{\cot(5x)}}$

599. $y = \frac{3 \sin(8x) \cos(8x)}{f \cdot g}$ $y' = 24 \cos(8x) \cos(8x) + 3 \sin(8x) \cdot -8 \sin(8x)$
 $f: 3 \cos(8x) \cdot 8$ $g': -8 \sin(8x)$
 $24 \cos(8x)$
 $y' = 24 \cos^2(8x) - 24 \sin^2(8x)$

600. $y = \frac{\ln x}{\sin x}$ $f': \frac{1}{x}$ $y' = \frac{\frac{1}{x} \sin(x) - \ln x \cdot \cos x}{\sin^2(x)}$
 $g': \cos x$
 $y' = \frac{\sin(x) - x \ln x \cdot \cos x}{x \cdot \sin^2(x)}$

$$601. \quad y = (\cos 3x)^2 - (\sin 3x)^2$$

$$y' = 2(\cos 3x)' \cdot -\sin 3x \cdot 3 - [2(\sin 3x)' \cdot \cos(3x) \cdot 3]$$

$$= 6 \cos(3x) \sin(3x) - 6 \cos(3x) \sin(3x)$$

$$\boxed{-12 \cos(3x) \sin(3x)}$$

$$602. \quad y = e^{\sin x} \Rightarrow y' = \boxed{e^{\sin(x)} \cdot \cos(x)}$$

$$603. \quad y = 3^{\cos x} \rightarrow y' = \underline{3^{\cos x} \cdot \ln 3 \cdot -\sin(x)}$$

$$604. \quad y = \log_3(\sin(2x))$$

$$y' = \frac{1}{\sin(2x) \cdot \ln 3} \cdot \cos(2x) \cdot 2 \Rightarrow \frac{2 \cos(2x)}{\ln 3 \cdot \sin 2x}$$

$$\boxed{\frac{2}{\ln 3} \cot(2x)}$$

$$605. \quad y = x e^{\ln(3x)}$$

Inverses!

$$\stackrel{\text{log}}{y} = x \cdot e^{\ln(3x)} \rightarrow y = x \cdot 3x \Rightarrow 3x^2 \rightarrow y' = 6x$$

$$606. \quad y = \frac{e^{3x}}{f} \cdot \frac{\tan(x)}{g}$$

$$f': 3e^{3x} \quad g': \sec^2(x)$$

$$y' = 3e^{3x} \tan(x) + e^{3x} \cdot \sec^2(x)$$

$$\boxed{e^{3x} (3 \tan(x) + \sec^2(x))}$$

607. $y = e^{\frac{1}{x^2}} = e^{x^{-2}}$
 ~~$y' = \frac{1}{e^{x^2}}$~~ $y' = e^{x^{-2}} \cdot -2x^{-3}$
 $\frac{-2e^{x^{-2}}}{x^3}$

608. $y = e^{x^2/4}$

~~$y = e^{\frac{1}{4}x^2}$~~ $\rightarrow y' = e^{\frac{1}{4}x^2} \cdot \frac{1}{2}x$

609. $y = \ln(\sec(x) + \tan(x))$

$y' = \frac{1}{\sec(x) + \tan(x)} \cdot (\sec(x)\tan(x) + \sec^2(x))$

$\frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}$

610. $y = \frac{x e^{\tan(x)}}{1}$ $y' = 1 \cdot e^{\tan(x)} + x \cdot e^{\tan(x)} \cdot \sec^2(x)$
 $f' = 1$ $g' = \frac{1}{e^{\tan(x)}} \rightarrow \sec^2(x)$
 $e^{\tan(x)} (1 + x \sec^2(x))$