Name_____ #3,4

Topic 1: Fractional & Negative Exponents

Simplify using only positive exponents

1. $-3x^{-3}$

2. $-5\left(\frac{3}{2}\right)(4-9x)^{\frac{-1}{2}}(-9)$

3. $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^2}\right]$

4. $\left(16x^2y\right)^{\frac{3}{4}}$

5. $-\frac{x^{\frac{-1}{2}}}{2}\sin\sqrt{x}$

6. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

$\frac{\sqrt[4]{4(x-4)}}{(x-4)^{3/4}}$

$\frac{2(x-4)^1}{(x-4)^{3/4}} \Rightarrow 2(x-4)^{1/4} \Rightarrow 2\sqrt[4]{x-4}$

7. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

8. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

9. $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{\frac{-1}{2}}$

$-4\left[\frac{1}{\left(\frac{2x-1}{2x+1}\right)^3}\right]\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

$-4\left[\frac{1}{\frac{(2x-1)^3}{(2x+1)^3}}\right]$

$-4\left[\frac{1}{1}\cdot\frac{(2x+1)^3}{(2x-1)^3}\right]\left[\frac{2\left(\cancel{(2x+1)}-\cancel{(2x-1)}\right)}{(2x+1)^2}\right]$

7

$-4\frac{(2x+1)^{\cancel{3}}}{(2x-1)^3}\cdot\frac{2(2)}{\cancel{(2x+1)^2}} \Rightarrow \frac{-16(2x+1)}{(2x-1)^3}$

#2, 8

Topic 2: Domain

Find the domain of the following functions:

1. $y = \dfrac{3x-2}{4x+1}$

2. $y = \dfrac{x^2-4}{2x+4}$

3. $y = \dfrac{x^2-5x-6}{x^2-3x-18}$ $\dfrac{(x-6)(x+1)}{(x-6)(x+3)}$

$x \neq 6, x \neq -3$

$(-\infty,-6)\cup(-6,-3)\cup(-3,\infty)$

4. $y = \dfrac{2^{2-x}}{x}$

5. $y = \sqrt{x-3} - \sqrt{x+3}$

$x-3 \geq 0$ $\quad x+3 \geq 0$

$x \geq 3$ $\quad x \geq -3$

$[3,\infty)$

6. $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7. $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

$x > -5$

$(-5,\infty)$

8. $y = \sqrt{x^2-5x-14}$

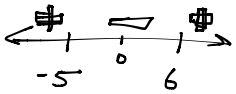
9. $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

$x^2-x-30 > 0$

$(x-6)(x+5) > 0$

$x = 6,\ x = -5$

$x=0$
$f(0) = -30$
neg



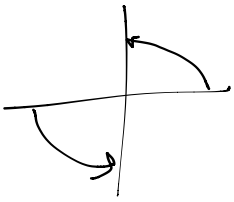
$x < -5$ $\quad x > 6$

$(-\infty,-5)\cup(6,\infty)$

10. $y = \log(2x-12)$

11. $y = \sqrt{\tan x}$

$\tan x \geq 0$



$[0, \pi/2)\cup[\pi, \frac{3\pi}{2})$

12. $y = \dfrac{x}{\cos x}$

#2, 7

Topic 3: Solving inequalities (absolute value)

Write the following absolute value expressions as piecewise expressions

1. $y = |2x - 4|$

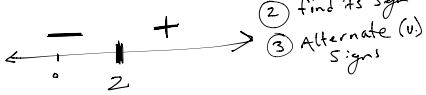
$2x - 4 = 0$

$x = 2$

Sign chart ① plug in a # on one side of the root

② find its sign

③ Alternate (+, −) signs



$x = 0$

$2(0) - 4 = -4 = \text{neg.}$

So:

$$f(x) = \begin{cases} 2x - 4, & \text{if } x \geq 2 \\ -(2x-4), & \text{if } x < 2 \end{cases}$$

2. $y = |6 + 2x| + 1$

3. $y = |4x + 1| + 2x - 3$

$4x + 1 = 0$

$x = -1/4$

$$f(x) = \begin{cases} 6x - 2, & \text{if } x \geq -1/4 \\ -2x - 4, & \text{if } x \leq -1/4 \end{cases}$$

Solve the following absolute value inequalities

4. $|x - 3| > 12$

5. $|x - 3| \leq 4$

$x - 3 \leq 4$

$x \leq 7$

$-(x-3) \leq 4$

$-x + 3 \leq 4$

$-x \leq 1$

$x \geq -1$

$-1 \leq x \leq 7$

6. $|10x + 8| > 2$

7. $|3x - 4| > -2$

8. $|x - 6| > -8$

9. $|x + 1| \leq |x - 3|$

#2, 6

Topic 4: Solving inequalities (quadratic)

Write the following absolute value expressions as piecewise expressions

1. $|x^2 - 1|$

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1, 1$

$+ \quad - \quad +$

$-1 \quad 1$

$$f(x) = \begin{cases} x^2-1 & \text{if } x \le -1 \\ -x^2+1 & \text{if } -1 < x < 1 \\ x^2-1 & \text{if } x \ge 1 \end{cases}$$

2. $|x^2 + x - 12|$

3. $|x^2 + 4x + 4|$

$[x+2][x+2]$

$x=-2$

$+ \quad + $

-2

$f(x) = x^2+4x+4$

Solve the following by factoring and making appropriate sign charts.

4. $x^2 - 16 > 0$

$(x+4)(x-4) > 0$

$+ \quad - \quad +$

$-4 \quad 0 \quad 4$

$(-\infty, -4) \cup (4, \infty)$

5. $x^2 + 6x - 16 > 0$

$(x+8)(x-2) > 0$

$+ \quad - \quad +$

$-8 \quad 0 \quad 2$

$(-\infty, -8) \cup (2, \infty)$

6. $x^2 - 3x \ge 10$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & -1 & -4 \\ & & -4 & 0 & 4 \\ \hline & 1 & 0 & -1 & \end{array}$$

7. $2x^2 + 4x \le 3$

$2x^2+4x-3 \le 0$

$x = \dfrac{-4 \pm \sqrt{16+4(2)(3)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{16+24}}{4}$

$x = \dfrac{-4 \pm \sqrt{40}}{4} \Rightarrow \dfrac{-4 \pm 2\sqrt{10}}{4}$

$x = \dfrac{-2 \pm \sqrt{10}}{2}$

$+ \quad - \quad +$

$\dfrac{-2-\sqrt{10}}{2} \quad 0 \quad \dfrac{-2+\sqrt{10}}{2}$

$\left(-\infty, \dfrac{-2-\sqrt{10}}{2}\right] \cup \left[\dfrac{-2+\sqrt{10}}{2}, \infty\right)$

8. $x^3 + 4x^2 - x \ge 4$

$x^3+4x^2-x-4 \ge 0$

$(x+4)(x^2-1) \ge 0$

$(x+4)(x+1)(x-1) \ge 0$

$x = -4, -1, 1$

$- \quad + \quad - \quad +$

$-4 \quad -1 \quad 0 \quad 1$

$[-4, -1] \cup [1, \infty)$

9. $2\sin^2 x \ge \sin x \qquad 0 \le x < 2\pi$

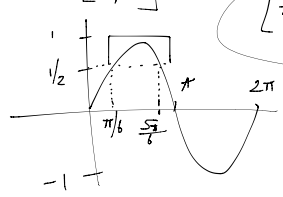
$2\sin^2 x - \sin x \ge 0$

$(\sin x)(2\sin x - 1) \ge 0$

$\sin x \ge 0 \qquad 2\sin x \ge 1$

$[0, \pi] \qquad \sin x \ge 1/2$

$[\pi/6, 5\pi/6]$



#11, 12

Topic 6: Function transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$:

1. $f(x) - 4$

translate down 4

2. $f(x-4)$

translate right 4

3. $-f(x+2)$

translate left 2, reflect across x-axis

4. $5f(x) + 3$

Stretch in y-direction, translate up 3

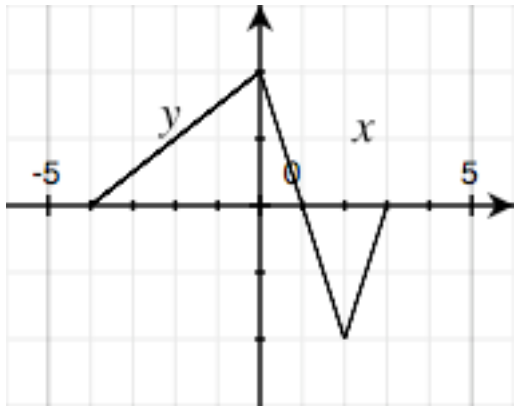
5. $f(2x)$

Compression in x-direction by 2

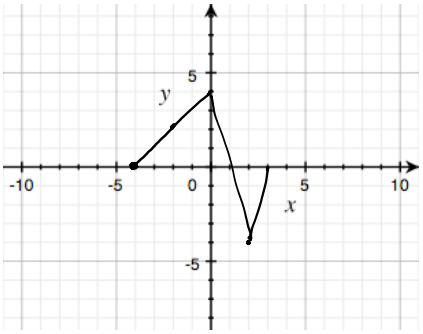
6. $|f(x)|$

all negative y-values reflected across x-axis

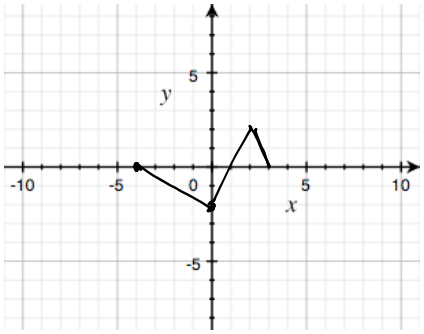
Here is a graph of $y = f(x)$. Sketch the following graphs



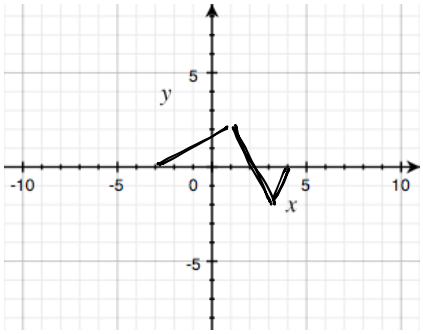
7. $y = 2f(x)$



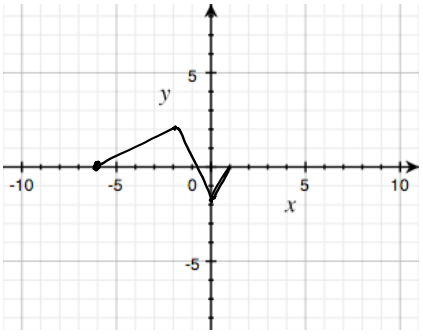
8. $y = -f(x)$



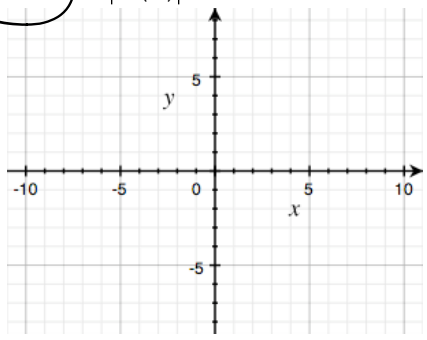
9. $y = f(x-1)$



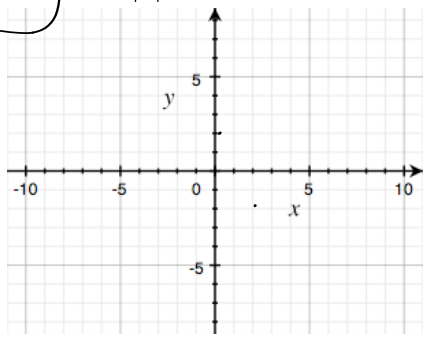
10. $y = f(x+2)$



11. $y = |f(x)|$



12. $y = f|x|$



4, 7

Topic 8: Even and odd functions

Show work to determine if the relation is even, odd, or neither

1. $f(x) = 2x^2 - 7$

$f(-x) = 2(-x)^2 - 7$

$2x^2 - 7$

so $f(-x) = f(x)$

$\boxed{\text{even}}$

2. $f(x) = -4x^3 - 2x$

$f(-x) = -4(-x)^3 - 2(-x)$

$= 4x^3 + 2x$

$f(-x) = -1(-4x^3 - 2x)$

$f(-x) = -f(x)$

so, odd

3. $f(x) = 4x^2 - 4x + 4$

$f(-x) = 4(-x)^2 - 4(-x) + 4$

$4x^2 + 4x + 4$

$\neq f(x)$ $\neq -f(x)$

neither

4. $f(x) = x - \dfrac{1}{x}$

5. $f|x| = |x| - x^2 + 1$

6. $5x^2 - 6y = 1$

7. $y = e^x - \dfrac{1}{e^x}$

8. $3y^3 = 4x^3 + 1$

9. $3x = |y|$

$f(x) = y = \begin{cases} 3x, & x \geq 0 \\ -3x, & x < 0 \end{cases}$

$f(-x) = \begin{cases} -3x, & x \geq 0 \\ 3x, & x < 0 \end{cases}$

$= \begin{cases} -1(3x), & x \geq 0 \\ -1[-3x], & x < 0 \end{cases}$

$= -f(x)$ $\boxed{\text{odd}}$

6, 7, 8

Topic 9: Solving quadratic equations and quadratic formula

Solve each equation

1. $7x^2 - 3x = 0$

$x(7x-3)=0$

$x=0 \qquad x=3/7$

2. $4x(x-2)-5x(x-1)=2$

$4x^2-8x-5x^2+5x=2$

$-x^2-3x-2=0$

$x^2+3x+2=0$

$(x+2)(x+1)=0$

$\boxed{x=-2 \quad x=-1}$

3. $x^2+6x+4=0$

$x=\frac{-6\pm\sqrt{36-4(1)(4)}}{2(1)}$

$\boxed{x=\frac{-6\pm\sqrt{20}}{2}}$

4. $2x^2-3x+3=0$

$x=\frac{3\pm\sqrt{9-4(2)(3)}}{4}$

$x=\frac{3\pm\sqrt{-13}}{4}$

$x=\frac{3}{4}\pm\frac{\sqrt{13}}{4}i$

5. $2x^2-(x+2)(x-3)=12$

$2x^2-(x^2-x-6)=12$

$2x^2-x^2+x+6-12=0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$\boxed{x=-3,2}$

6. $x+\frac{1}{x}=\frac{13}{6}$

7. $x^4-9x^2+8=0$

8. $x-10\sqrt{x}+9=0$

9. $\left(\frac{1}{x^2}-\frac{1}{x}=6\right)x^2$

$1-x=6x^2$

$0=6x^2+x-1$

$x=\frac{-1\pm\sqrt{36-4(6)(-1)}}{2(6)}$

$\boxed{x=\frac{-1\pm\sqrt{60}}{12}}$

$x=\frac{-1\pm2\sqrt{15}}{12}$

Topic 10: Asymptotes

#6

For each function, find the equations of both the vertical asymptote(s) and horizontal asymptotes (if they exist)

1. $y = \dfrac{x}{x-3}$

V.A. $x=3$

H.A. $y=1$

2. $y = \dfrac{x+4}{x^2-1}$

V.A. $x=1,\ x=-1$

H.A. $y=0$

3. $y = \dfrac{x+4}{x^2+1}$

V.A. n/a

H.A. $y=0$

4. $y = \dfrac{x^2-2x+1}{x^2-3x-4}$

$(x-4)(x+1)=0$

V.A. $x=4,\ x=-1$

H.A. $y=1$

5. $y = \dfrac{x^2-9}{x^3+3x^2-18x}$

$x(x^2+3x-18)=0$

$x(x+6)(x-3)=0$

V.A. $x=0,\ x=-6,\ x=3$

H.A. $y=0$

6. $y = \dfrac{2x^2+6x}{x^3-3x^2-4x}$

7. $y = \dfrac{x^2-x-6}{x^3-x^2+x-6}$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -6 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & - \end{array}$$

$(x-2)(x^2+x+3)=0$

V.A. $x=2$

H.A. $y=0$

8. $y = \dfrac{2x^3}{x^3-1}$

$(x-1)(x^2+x+1)=0$

V.A. $x=1$

H.A. $y=2$

9. $y = \dfrac{\sqrt{x}}{2x^2-10}$ $x^{1/2}$

$2(x^2-5)=0$

$x^2=5$

V.A. $x=\pm\sqrt{5}$

H.A. $y=0$