1.19 Sample A.P. Problems on Limits

328. For the function $f(x) = \dfrac{2x-1}{|x|}$, find the following:

a) $\lim\limits_{x\to\infty} f(x)$;

b) $\lim\limits_{x\to-\infty} f(x)$;

c) $\lim\limits_{x\to 0^+} f(x)$;

d) $\lim\limits_{x\to 0^-} f(x)$;

e) All horizontal asymptotes;

f) All vertical asymptotes.

329. Consider the function $h(x) = \dfrac{1}{1-2^{1/x}}$.

a) What is the domain of h?

b) Find all zeros of h.

c) Find all vertical and horizontal asymptotes of h.

d) Find $\lim\limits_{x\to 0^+} h(x)$.

e) Find $\lim\limits_{x\to 0^-} h(x)$.

f) Find $\lim\limits_{x\to 0} h(x)$.

330. Consider the function $g(x) = \dfrac{\sin|x|}{x}$ defined for all real numbers.

a) Is $g(x)$ an even function, an odd function, or neither? Justify your answer.

b) Find the zeros and the domain of g.

c) Find $\lim\limits_{x\to 0} g(x)$.

331. Let $f(x) = \begin{cases} \sqrt{1-x^2} & 0 \le x < 1 \\ 1 & 1 \le x < 2 \\ 2 & x = 2 \end{cases}$.

a) Draw the graph of f.

b) At what points c in the domain of f does $\lim\limits_{x\to c} f(x)$ exist?

c) At what points does only the left-hand limit exist?

d) At what points does only the right-hand limit exist?