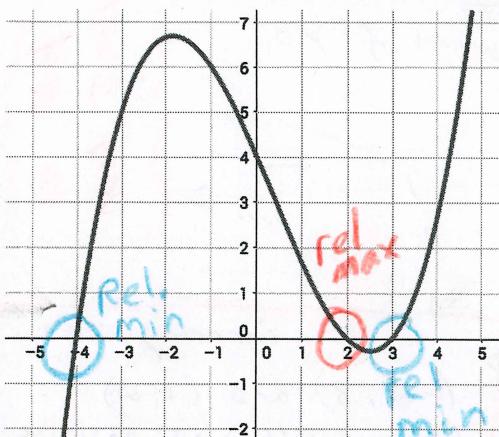


D-AD7

Practice Assessment

S25

Given below is the graph of f' the first derivative of f . Use it to answer #1 and 2.



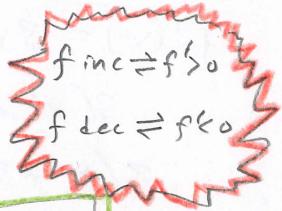
1. Over what interval(s) is f decreasing? Explain in detail.

means f' is negative

$(-\infty, -4)$ and

$(2, 3)$. f' is negative

here, so f has neg. slope.



2. Where, if anywhere, does f achieve a relative minimum? Justify your response.

f has relative minima

at $x = -4$ and $x = 3$.

because $f' = 0$ there

and changes from negative to positive.

f : dec \rightarrow cn \rightarrow increasing

f' : neg \rightarrow zero \rightarrow positive unde

Rel. max:

f : Inc \rightarrow cn \rightarrow dec.

f' : pos \rightarrow zero or unde \rightarrow neg.

D-AD8

3. Find the absolute extrema of $f(x) = 2x^3 - 6x - 2$ over the interval $[-4, 0]$.

① Critical Numbers

$$f'(x) = 6x^2 - 6 = 0$$

$$6(x^2 - 1) = 0$$

$$6(x+1)(x-1) = 0$$

$$\begin{matrix} \downarrow & \downarrow \\ x = -1 & x = 1 \end{matrix}$$

C.N.

② Try C.N. and Endpts.

$$f(-1) = 2(-1)^3 - 6(-1) - 2$$

$$-2 + 6 - 2 = \underline{\underline{2}} \leftarrow \text{biggest}$$

$$f(1) = 2(1)^3 - 6(1) - 2$$

$$2 - 6 - 2 = \underline{\underline{-6}}$$

$$f(-4) = 2(-4)^3 - 6(-4) - 2 = \underline{\underline{-106}} \leftarrow \text{smallest}$$

$$f(0) = 2(0)^3 - 6(0) - 2 = \underline{\underline{-2}}$$

ABS. MAX

(-1, 2)

ABS. MIN

(-4, -106)

4. Find and classify all relative maxima and relative minima of $f(x) = -x^3 + 3x^2 + 2$. Justify your classifications.

must occur @

Critical numbers, but
need to see sign change!

C.N.?

$$f'(x) = -3x^2 + 6x = 0$$

$$-3x(x-2) = 0$$

$$\checkmark \quad \checkmark$$

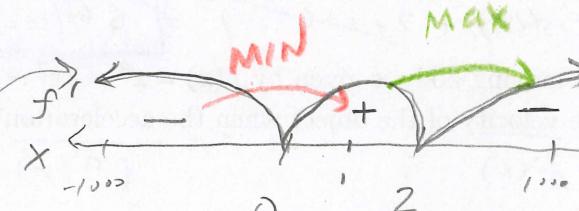
$$-3x = 0$$

$$x-2 = 0$$

$$\underline{x=0}$$

$$\underline{x=2}$$

C.N.



$$f'(x) = -3x(x-2)$$

$$\bullet -1000 \rightarrow (+)(-) = -$$

$$\bullet 1 \rightarrow (-)(-) = +$$

$$\bullet 1000 \rightarrow (-)(+) = -$$

$x=0$ is a rel. min. because

$f'(0) = 0$ and sign change

neg \rightarrow pos $x=2$ is a rel. max

because $f'(2) = 0$ and sign

change pos \rightarrow neg.

① Plot C.N. on number line

② Determine sign in intervals by plugging test values into f'

D-AD9

5. For what interval(s) is the function $f(x) = x^3 + 3x^2 - 9x + 7$ increasing? Justify your answer.

Find C.N.

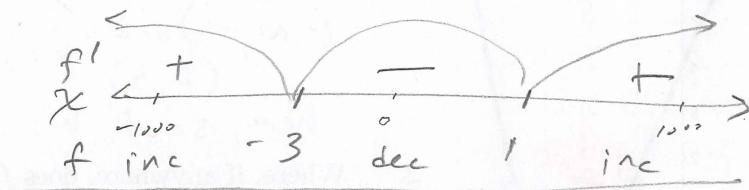
$$f'(x) = 3x^2 + 6x - 9 = 0$$

$$3(x^2 + 2x - 3) = 0$$

$$3(x+3)(x-1) = 0$$

$$x = -3 \quad x = 1$$

C.N.



$$f'(-\infty) = 3(-)(-) = +$$

$$f'(0) = 3(3)(-1) = -$$

$$f'(1) = 3(+)(+) = +$$

f is increasing from $(-\infty, -3)$ and $(1, \infty)$ because $f'(x)$ is positive there.

Very Similar
to #4

D-CD8

6. Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x) = x^3 - 2x^2$ over $[0, 2]$.

Avg.

$$\frac{f(2) - f(0)}{2 - 0} = \frac{2^3 - 2(2)^2 - 0}{2 - 0} = \frac{0 - 0}{2} = 0$$

Avg. = Inst.

differentiable b/c
polynomial

Inst.

$$f'(x) = 3x^2 - 4x$$

MVT.

$$0 = 3x^2 - 4x$$

$$0 = x(3x - 4)$$

$$x = 0 \leftarrow \text{not in } (0, 2) \\ x = \frac{4}{3}$$

7. Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x) = \sqrt{x}$ over $[0, 4]$

$$\frac{f(4) - f(0)}{4 - 0} = \frac{2 - 0}{4} = \frac{1}{2} \rightarrow \text{MVT.}$$

$$\text{Inst: } f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

D-AD17

8. The position, in feet, of a particular moving body is modeled as a differentiable function of time, in seconds, by $s(t) = 3 \sin 2t - 4 \cos 5t$. Find the initial velocity and acceleration of the object. Include units in your answer.

VEL

$$s'(t) = v(t) = 3 \cos(2t) \cdot 2 - 4 \cdot -\sin(5t) \cdot 5$$

$$v(t) = 6 \cos(2t) + 20 \sin(5t)$$

$$\text{Initial: } v(0) = 6 \cos(0) + 20 \sin(0) = 6 \text{ ft/sec}$$

$t=0$

pos → acc
 $s \quad s' \quad s''$

$$ACC \quad s''(t) = v'(t) = a(t)$$

$$= 6 \cdot \sin(2t) \cdot 2 + 2 \cdot \cos(5t) \cdot 5$$

$$a(t) = 12 \sin(2t) + 100 \cos(5t)$$

$$a(0) = 12 \sin(0) + 100 \cos(0)$$

100
ft/sec

a

9. The position of a moving body is given by $f(x) = x^3 - 2x^2 - 5x - 1$ where f is in meters and x is in seconds. Find the velocity of the object when the acceleration is 0. Include units in your answer.

$$f'(x)$$

$$f''(x) = 0$$

$$f'(x) = 3x^2 - 4x - 5$$

$$f''(x) = 6x - 4 = 0$$

$$6x = 4$$

$$x = \frac{2}{3} \text{ sec.}$$

Velocity:

$$f'\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) - 5$$

$$f'\left(\frac{2}{3}\right) = -\frac{19}{3}$$

m/s

calc.