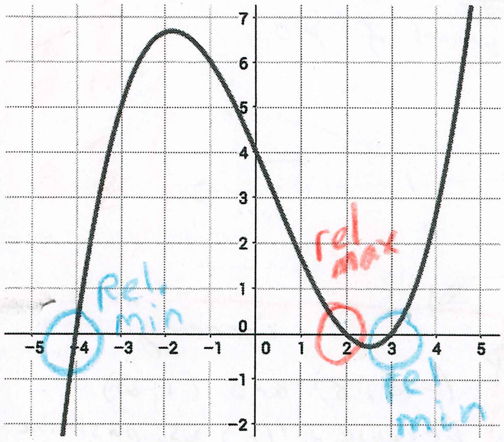
Sols

D-AD7

<u>Practice Assessment</u>

Given below is the graph of f' the first derivative of f. Use it to answer #1 and 2.



1. Over what interval(s) is f decreasing? Explain in detail.

means f' is negative ✓

$(-\infty, -4)$ and $(2, 3)$. f' is negative here, so f has neg. slope.

f inc $\rightleftharpoons f' > 0$

f dec $\rightleftharpoons f' < 0$

2. Where, if anywhere, does f achieve a relative minimum? Justify your response.

f has relative minima @ $x = -4$ and $x = 3$. because $f' = 0$ there and changes from negative to positive.

f: dec → C.N. → increasing

f': neg → zero, undef. → positive

Rel. max:

f: Inc → C.N. → dec.

f': pos → zero or undef. → neg.

D-AD8

3. Find the absolute extrema of $f(x) = 2x^3 - 6x - 2$ over the interval [-4,0].

① Critical Numbers

$f'(x) = 6x^2 - 6 = 0$

$6(x^2 - 1) = 0$

$6(x+1)(x-1) = 0$

$\underbrace{x = -1 \quad x = 1}_{C.N.}$

② Try C.N. and Endpts.

$f(-1) = 2(-1)^3 - 6(-1) - 2$
$-2 + 6 - 2 = \underline{2}$ ← biggest

$f(1) = 2(1)^3 - 6(1) - 2$
$2 - 6 - 2 = \underline{-6}$

$f(-4) = 2(-4)^3 - 6(-4) - 2$
$-128 + 24 - 2 = \underline{-106}$ ← smallest

$f(0) = 2(0)^3 - 6(0) - 2 = \underline{-2}$

ABS. MAX $(-1, 2)$

ABS. MIN $(-4, -106)$

4. Find and classify all relative maxima and relative minima of $f(x) = -x^3 + 3x^2 + 2$. Justify your classifications.

↳ must occur @ Critical numbers, but need to see sign change!

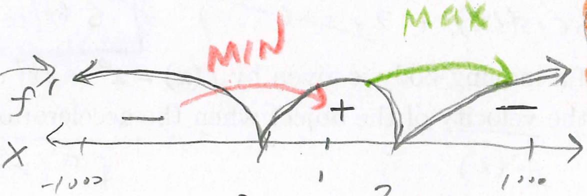
C.N.?

$f'(x) = -3x^2 + 6x = 0$

$-3x(x-2) = 0$

$-3x = 0 \qquad x - 2 = 0$

$\underbrace{x = 0 \qquad x = 2}_{C.N.}$



$f'(x) = -3x(x-2)$

- $-1000 \to (+)(-) = -$
- $1 \to (-)(-) = +$
- $1000 \to (-)(+) = -$

① Plot C.N. on number line

② Determine sign in intervals by plugging test values into f'

$x = 0$ is a rel. min. because $f'(0) = 0$ and sign change neg → pos. $x = 2$ is a rel. max because $f'(2) = 0$ and sign change pos → neg.

D-AD9

5. For what interval(s) is the function $f(x) = x^3 + 3x^2 - 9x + 7$ increasing? Justify your answer.

means $f' > 0$

Very Similar to #4

Find C.N.

$f'(x) = 3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$x = -3 \quad x = 1$

C.N.

f'	$+$		$-$		$+$
x	-1000	-3	0	1	1000
f	inc		dec		inc

$f'(-1000) = 3(-)(-) = +$

$f'(0) = 3(3)(-1) = -$

$f'(1000) = 3(+)(+) = +$

f is increasing from $(-\infty, 3)$ and $(1, \infty)$ because $f'(x)$ is positive there.

D-CD8

6. Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x) = x^3 - 2x^2$ over $[0,2]$.

Avg. = Inst.

differentiable b/c polynomial

Avg

$\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{2^3 - 2(2)^2 - 0}{2 - 0} = \dfrac{0 - 0}{2} = 0$

Inst.

$f'(x) = 3x^2 - 4x$

MVT.

$0 = 3x^2 - 4x$

$0 = x(3x - 4)$

$x = 0 \leftarrow$ Not in $(0,2)$!

$x = \frac{4}{3}$

7. Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x) = \sqrt{x}$ over $[0,4]$

Avg. $\dfrac{f(4) - f(0)}{4 - 0} = \dfrac{2 - 0}{4} = \dfrac{1}{2}$

Inst: $f'(x) = \frac{1}{2} x^{-1/2} = \dfrac{1}{2x^{1/2}} = \dfrac{1}{2\sqrt{x}}$

MVT.

$\dfrac{1}{2} = \dfrac{1}{2\sqrt{x}} \rightarrow 2\sqrt{x} = 2$

$(\sqrt{x} = 1)^2$

$x = 1 \leftarrow$ is in $(0,4)$

D-AD17

8. The position, in feet, of a particular moving body is modeled as a differentiable function of time, in

8. Use a tangent line to approximate $\sqrt[3]{122}$

- fixed point: $(125, 5)$
  (x_1, y_1)
- function: $f(x) = \sqrt[3]{x} = x^{1/3}$

$y - 3 = m(x - 125)$

$m?$ $f'(125)$ $\longrightarrow$ $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$= \frac{1}{3(\sqrt[3]{x})^2}$

$f'(125) = \frac{1}{3(\sqrt[3]{125})^2}$

$\frac{1}{3(5)^2} = \frac{1}{3 \cdot 25} = \frac{1}{75}$

$y - 5 = \frac{1}{75}(x - 125)$

$x = 122$

$y - 5 = \frac{1}{75}(122 - 125)$

$y - 5 = \frac{1}{75}(-3)$

$y - 5 = -\frac{3}{75}$

$y = 5 - \frac{3}{75}$

$\boxed{y = 4\frac{72}{75}}$

L'Hôpital's Rule

If $\lim\limits_{x\to c}\frac{f(x)}{g(x)}$ yields an indeterminate form, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

9. $\lim\limits_{t\to\infty}\frac{3t^2}{3^t} = \frac{3\infty}{3^\infty} = \frac{\infty}{\infty}$

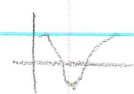
l'hop

$\lim\limits_{t\to\infty}\frac{6t}{3^t\cdot\ln 3} = \frac{\infty}{\infty}$

l'hop

$\lim\limits_{t\to\infty}\frac{6}{3^t\ln 3\cdot\ln 3} = \frac{6}{3^\infty\ln 3\cdot\ln 3} = \frac{6}{\infty} = 0$

10. $\lim\limits_{x\to 3}\frac{\sin(\pi x)}{x^2-9} = \frac{0}{0}$



l'hop

$3\pi \longrightarrow \pi$

$\lim\limits_{x\to 3}\frac{\pi\cdot\cos(\pi x)}{2x} \Rightarrow \frac{\pi\cdot\overbrace{\cos(3\pi)}^{-1}}{2(3)} = -\frac{\pi}{6}$

11. $\left.\frac{dy}{dx}\right|_{(1,2)}$ when $xy^2 + 2xy = 8$

$$\frac{d}{dx}\left[\underbrace{xy^2}_{\text{product rule}} + \underbrace{2xy}_{\text{product rule}}\right] = [8]\frac{d}{dx}$$

$$\frac{d}{dx}[f \cdot g] = f'g + fg'$$

$\underset{f}{x}\,\underset{g}{y^2}$

$f: x \quad g: y^2$

$f': 1 \quad g': 2y \cdot y'$

$\underset{f}{2x}\,\underset{g}{y}$

$f: 2x \quad g: y$

$f': 2 \quad g': 1y'$

$$1 \cdot y^2 + x \cdot 2y \cdot y' + 2 \cdot y + 2x \cdot y' = 0$$

$$y^2 + 2xy\,\underline{y'} + 2y + 2x\underline{y'} = 0$$

$$2xy\,y' + 2xy' = -y^2 - 2y$$ ← send non y' terms out

$$y'(2xy + 2x) = -y^2 - 2y$$ ← factor out y'

← divide

$$y' = \frac{-y^2 - 2y}{2xy + 2x}$$

$-4/3$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{-(2)^2 - 2(2)}{2(1)(2) + 2(1)} = \frac{-4-4}{4+2} = \frac{-8}{6}$$

x y