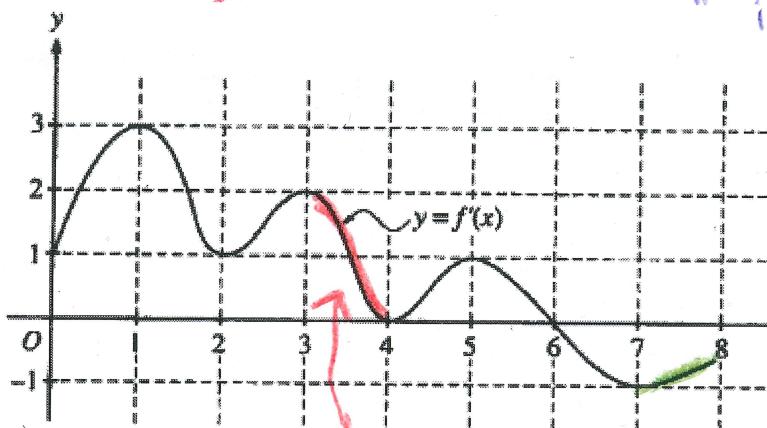


D-AD10

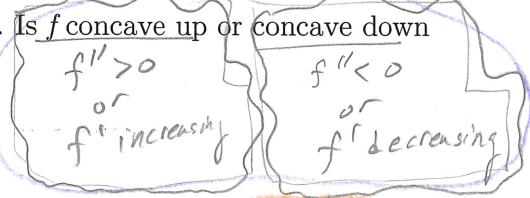
## Practice Assessment

Solutions

1. Shown here is the graph of the first derivative of some function  $f(x)$ . Is  $f$  concave up or concave down over the interval  $(3, 4)$ ? Justify your response.



main idea



$f$  is concave down over  $(3, 4)$  because  $f'$  is decreasing [or,  $f''$  is negative].

ANSWER

$f'$  is decreasing  $\rightarrow f''$  negative  $\rightarrow f$  concave down.

2. Using the above graph, give an interval for which  $f$  is decreasing but concave up.

$f'$  neg       $f'$  increasing

So, below x-axis, but uphill.

(7, 8)

For 3 and 4, refer to the function  $f(x) = 5x^4 - x^5$ . Calculations/analysis do not need to be repeated if you need to make reference to the other problem.

D-AD11

$$f''(x) = 0 + \text{sign change}$$

3. Find any inflection points for the function  $f(x)$ . Justify your response.

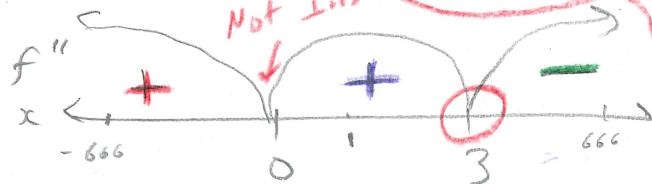
$$f'(x) = 20x^3 - 5x^4$$

$$f''(x) = 60x^2 - 20x^3 = 0$$

$$20x^2(3-x) = 0$$

$x=0$        $x=3$

T.P.



$$f''(1) = 20(1)(2) = +$$

$$f''(-666) = 20(+)(+) = +$$

$$f''(666) = 20(+)(-) = -$$

D-AD12

4. Find the interval(s) over which  $f(x)$  is concave up. Justify your response.

(See above sign chart/number line.)

$f(x)$  is concave up over  $(-\infty, 0) \cup (0, 3)$

because  $f''(x)$  is positive there.

D-AD13

5. Find any intervals for which  $f(x) = 2x^4 - 4x^2 - 3$  is decreasing and concave up. Show the calculations that lead to your conclusion.

Incl/Dec  $f'(x) = 8x^3 - 8x = 0$

$$8x(x^2 - 1) = 0$$

$$(8x)(x+1)(x-1) = 0$$

$$\begin{matrix} x=0 \\ \downarrow \\ x=-1 \\ \downarrow \\ x=1 \end{matrix}$$

C.N.

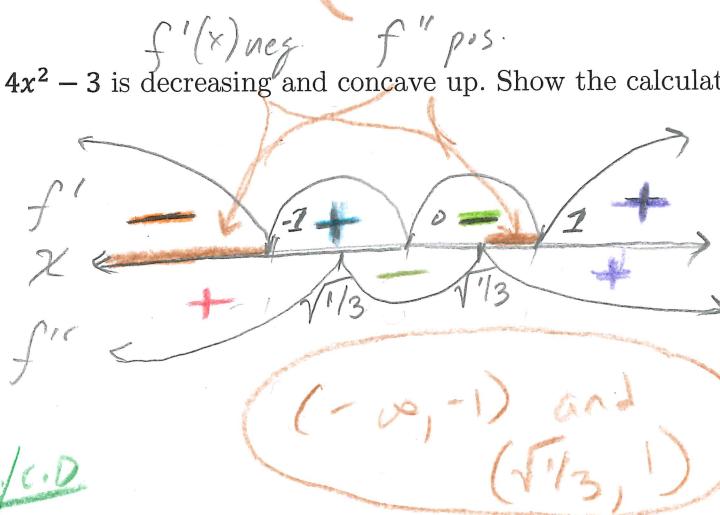
testing

$$f'(-\infty) = (-)(-)(-) = -$$

$$f'(-1/2) = (-)(+)(-) = +$$

$$f'(1/2) = (+)(+)(+) = -$$

$$f'(\infty) = (+)(+)(+) = +$$



testing

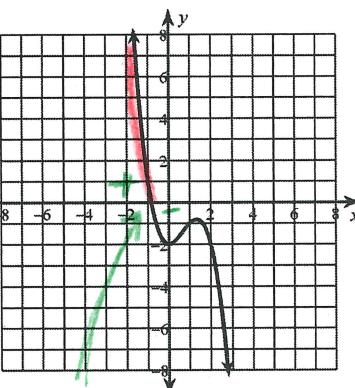
$$f''(-\infty) = 8(+/-1) = +$$

$$f''(0) = 8(-) = -$$

$$f''(\infty) = 8(\infty^2 - 1) = +$$

$$x = \pm \sqrt{1/3} \approx \pm 0.577$$

D-AD7



rel max

$f$  is increasing over  $(-\infty, -1)$  b/c

$f'$  is positive there

6. Shown here is  $f'(x)$  the first derivative of  $f(x)$ . Give any interval(s) where  $f(x)$  is increasing. Justify.

No,  $f'$  needs to change sign from negative to positive, which does not happen in this graph of  $f$ !

7. Does  $f(x)$  have any relative minima? If so, give the x-coordinate. If not, explain why not.

8. Find the absolute extrema for  $f(x) = -x^3 + 2x^2 + 4$  on the interval  $[-1, 1]$

[See Next page]

9. Find and classify any relative extrema. Justify your classifications.  $f(x) = -x^4 + 2x^2 - 4$

D-AD9

10. For what interval(s) is the function  $f(x) = x^3 + 3x^2 - 9x + 7$  increasing? Justify your answer.

## D-AD8

8. Find the absolute extrema for  $f(x) = -x^3 + 2x^2 + 4$  on the interval  $[-1, 1]$

$$\left\{ \begin{array}{l} \text{(1) Find C.N.} \\ \text{(2) Plug C.N. and} \\ \text{Endpoints into } f \\ \text{[Not } f' \text{]} \end{array} \right\}$$

$$f'(x) = -3x^2 + 4x = 0$$

$$-x(3x - 4) = 0$$

$$\downarrow \quad \downarrow$$

$$x=0 \quad x=\frac{4}{3} \quad \text{Not in } [-1, 1]!$$

C.N.

test  
 $f(0) = 4$  min

$$f(-1) = 7 \text{ max}$$

$$f(1) = 5$$

ABSOLUTE MAX:  
 $(-1, 7)$

ABSOLUTE MIN:  
 $(0, 4)$

9. Find and classify any relative extrema. Justify your classifications.  $f(x) = -x^4 + 2x^2 - 4$

$$f'(x) = -4x^3 + 4x = 0$$

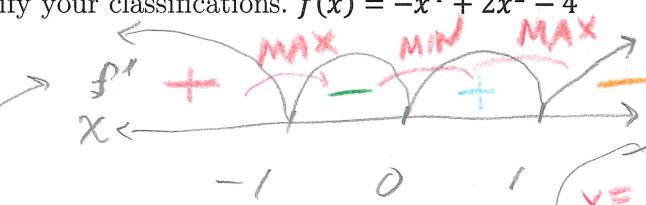
$$-4x(x^2 - 1) = 0$$

$$-4x(x+1)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$x=0 \quad x=-1 \quad x=1$$

C.N.



testing

$$f'(-\infty) = +(-)(-) = +$$

$$f'(-1/2) = +(+)(-) = -$$

$$f'(1/2) = -(+)(-) = +$$

$$f'(\infty) = -(+)(+) = -$$

$x = -1$  :  $x = 1$   
 rel. MAX  
 because  $f'(x)$   
 changes from  $+ \rightarrow -$

$x = 0$   
 rel. min  
 because  $f'(x)$   
 changes from  $- \rightarrow +$

## D-AD9

10. For what interval(s) is the function  $f(x) = x^3 + 3x^2 - 9x + 7$  increasing? Justify your answer.

$$f'(x) = 3x^2 + 6x - 9 = 0$$

$f'(x)$  positive

$$3(x^2 + 2x - 3) = 0$$

$$3(x+3)(x-1) = 0$$

$$\downarrow \quad \downarrow$$

$$x=-3 \quad x=1$$

C.N.



$f$  is increasing  
 over  $(-\infty, -3)$  and  $(1, \infty)$   
 b/c  $f'(x)$  is positive  
 there.

testing

$$f'(-\infty) = 3(-)(-) = +$$

$$f'(0) = -9 = -$$

$$f'(\infty) = 3(+)(+) = +$$