

Sols

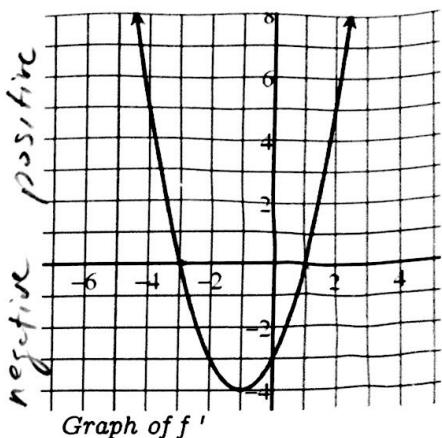
D-AD7 Practice Assessment

1. Given here is the graph of the derivative of  $f$ . Over what interval is  $f$  increasing? Justify your answer. (5)

$f$  is increasing where  $f'$  is positive  
 $f$  is decreasing where  $f'$  is negative

$f$  is increasing over  $(-\infty, -3)$  and  $(1, \infty)$

because  $f'$  is positive over these intervals.



D-AD8

2. Find the absolute extrema of  $f(x) = 2x^3 - 6x - 2$  over the interval  $[-4, 0]$ .

→ Find critical numbers  
 → Plug them and endpoints into  $f(x)$

$$f'(x) = 6x^2 - 6 = 0$$

$$6(x^2 - 1) = 0$$

$$6(x+1)(x-1) = 0$$

$$\begin{array}{|c|c|} \hline x = -1, & x = 1 \\ \hline \text{C.N.} & \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline x = 1 & \text{Not in } [-4, 0] \\ \hline \end{array}$$

Endpoints  
 $f(-4) = 2(-4)^3 - 6(-4) - 2$   
 $= -106$  min

$$f(0) = -2$$

C.N.  
 $f(-1) = 2(-1)^3 - 6(-1) - 2$   
 $= 2$  max

|                             |                                |
|-----------------------------|--------------------------------|
| <u>Abs Max</u><br>$(-1, 2)$ | <u>Abs Min</u><br>$(-4, -106)$ |
|-----------------------------|--------------------------------|

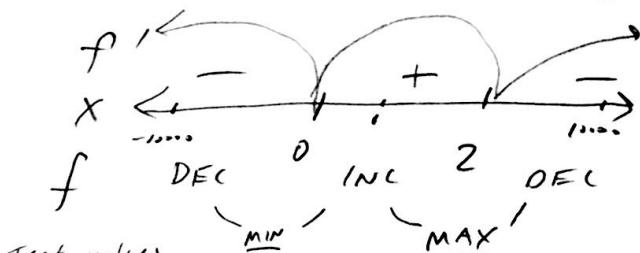
3. Find and classify all relative maxima and relative minima of  $f(x) = -x^3 + 3x^2 + 2$ . Justify.

→ Find C.N.  
 → use number line  
 → test values in  $f'(x)$ .

$$f'(x) = -3x^2 + 6x = 0$$

$$-3x(x-2) = 0$$

$$\begin{array}{|c|c|} \hline x = 0 & x = 2 \\ \hline \text{C.N.} & \\ \hline \end{array}$$



| Test values | $f'(1) = +$ | $f'(-\dots) = -$ | $f'(1\dots) = -$ | $f$ has rel. max @ $x=2$ b/c $f'$ changes sign $\leftarrow - \rightarrow +$ | $f$ has rel. min @ $x=0$ b/c $f'$ changes sign $- \rightarrow +$ |
|-------------|-------------|------------------|------------------|---|--|
|             |             |                  |                  |   |  |

- Number line
- ① Place Critical Numbers on it
  - ② Bungee hops to show intervals
  - ③ pick test values in each interval
  - ④ plug test values into  $f'(x)$
  - ⑤ Determine +/− of output
  - ⑥ Mark  $f$  inc or dec. as appropriate

## D-AD9

4. Over what intervals is  $f(x) = x^3 - 3x + 2$  increasing and over what intervals is it decreasing? Justify your response.

Very  
similar  
to  
#3  
and  
#1

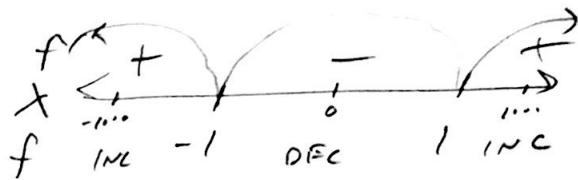
$$f'(x) = 3x^2 - 3 = 0$$

$$3(x^2 - 1) = 0$$

$$3(x+1)(x-1) = 0$$

$$\begin{cases} x = -1 \\ x = 1 \end{cases}$$

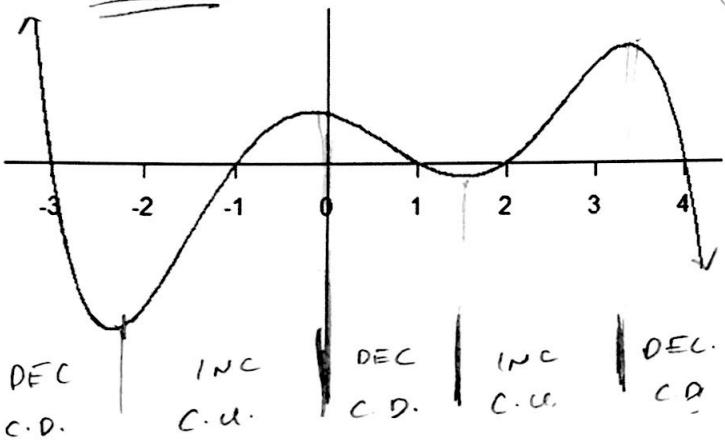
C-N



$f$  is increasing over  $(-\infty, -1)$  and  $(1, \infty)$  b/c  $f'$  is positive over these intervals.

## D-AD10

5. Shown here is the graph of the first derivative of a function  $f(x)$ . Find the intervals where  $f(x)$  is concave up and where it is concave down.



Concave up:  $f''$  is positive, which means  $f'$  is increasing

Concave down:  $f''$  is negative, which means  $f'$  is decreasing

$f$  is concave up:  $(-2, 0)$ ,  $(1.5, 3.5)$  because  $f'$  is increasing here.

$f$  is concave down:  $(-\infty, -2)$ ,  $(0, 1.5)$  because  $f'$  is decreasing here.  $(3.5, \infty)$

D-AD11] → will test next week

6. Find the location of all inflection points for the function  $f(x) = -x^3 + 4x^2 - 2$ . Justify your answer.

$$f'' = 0$$

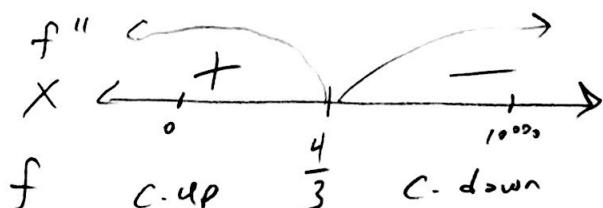
$$f' = -3x^2 + 8x$$

$$\text{f'' changes sign } f'' = -6x + 8 = 0$$

$$-6x = -8$$

$$x = \frac{8}{6} = \frac{4}{3}$$

terrace point



$x = \frac{4}{3}$  is an inflection point for  $f$  because  $f''$  changes sign here.