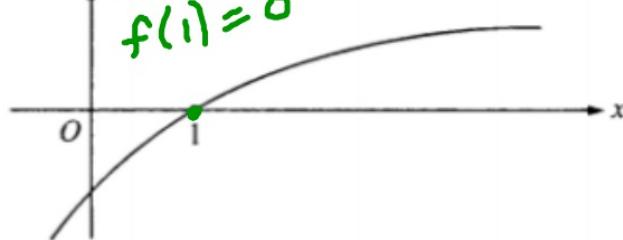


Good afternoon: please do warm up in notebooks

$$f''(1) < 0 \quad [C.P]$$

$$f'(1) > 0 \quad [Inc.]$$

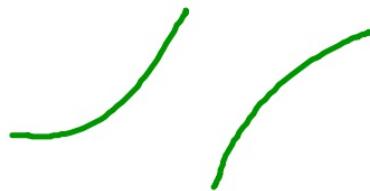
$$f(1) = 0$$



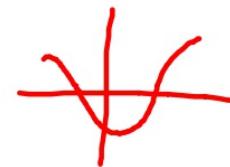
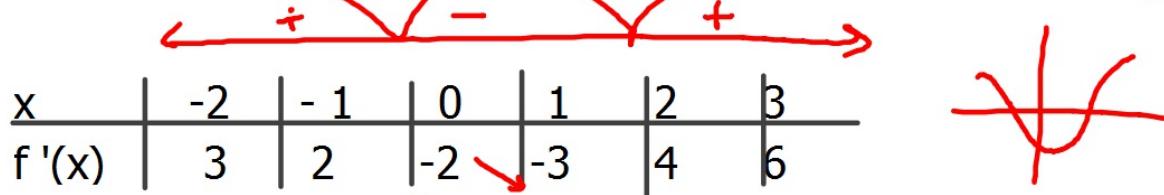
Rank the following from least to greatest

$$f''(1) < f(1) < f'(1)$$

graph of $f(x)$



Let F be a twice-differentiable function. Selected values of f' are given below



Explain why f must have a relative extrema between -1 and 0 . What type is it?

Sign change for f'

$+ \rightarrow -$ means max.

Between 0 and 1 , is f concave up or concave down? Explain.

$f'' < 0$ because f' is decreasing

Where do you think f has an inflection point? Explain.

$$\begin{array}{c|cc} x & 1 & 2 \\ \hline f' & -3 & 4 \end{array}$$

$f''(x) = 0$ between 1 and 2 because f' changes
from dec to inc.

HW

15. CU: $(2, \infty)$
CD: $(-\infty, 2)$
 $x=2$ IP

16. CU: $(-\infty, 2)$
CD: $(2, \infty)$
 $x=2$ IP

17. CU: $(-\infty, -2) \cup (0, \infty)$
CD: $(-2, 0)$
 $x=-2, x=0$ IP

18. CD: everywhere
no IP

19. CU: $(-\infty, 2) \cup (4, \infty)$
CD: $(2, 4)$
 $x=2, x=4$ IF

20. CU: $(-\infty, 1.5) \cup (2, \infty)$
CD: $(1.5, 2)$
 $x=1.5, x=2$ IF

21. CD: $(-3, \infty)$
no IP

22. CD: $(-\infty, 9)$
no IP

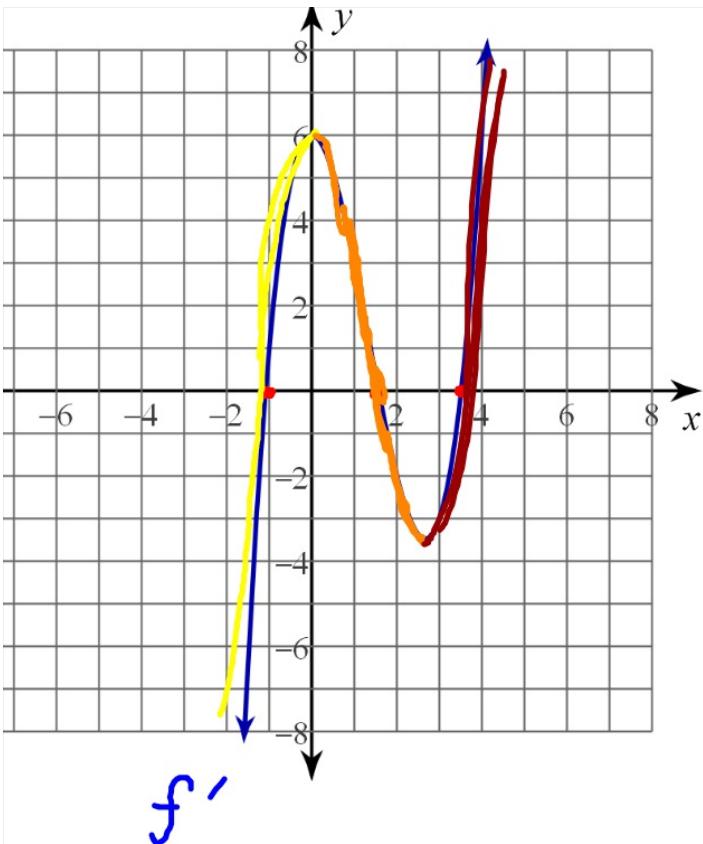
23

24. CU: $(0, 9)$
CD: $(9, \infty)$
 $x=9$ IP

Concave upward: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$

Concave downward: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

Points of inflection: $\left(-\frac{\sqrt{3}}{3}, 3\right)$ and $\left(\frac{\sqrt{3}}{3}, 3\right)$



Example:

Here is the first derivative of $f(x)$. Over what intervals is f concave up and concave down?

C.U. $(-\infty, 0)$: f' is increasing
 $(2.5, \infty)$ $[f'' > 0]$

C.D. $(0, 2.5)$: f' is decreasing
 $[f'' < 0]$

$$y = \frac{2x}{x+1}$$

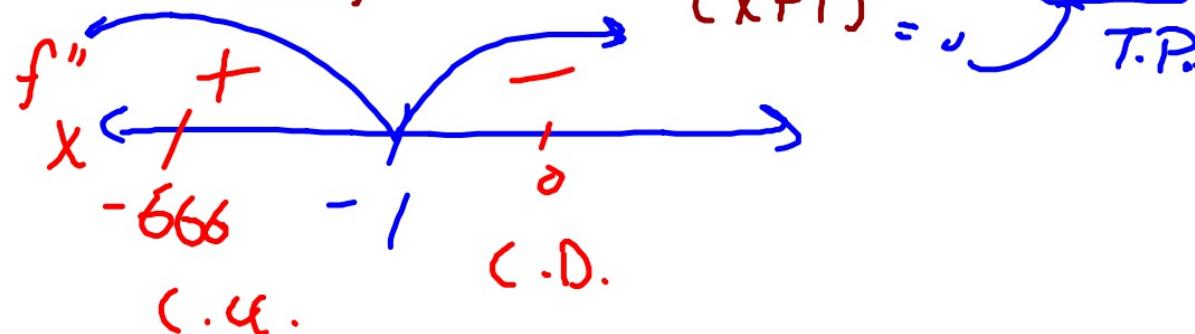
Find any interval(s) over which the function is concave up.

$$\boxed{f'' > 0}$$

$$y' = \frac{2}{(x+1)^2} \quad f \quad g \quad f' = 0$$
$$g' = 2(x+1)' \cdot 1$$

$$y'' = \frac{0 - 2 \cdot 2(x+1)' \cdot 1}{(x+1)^4} \rightarrow \frac{-4x - 4}{(x+1)^4} = 0 \rightarrow$$
$$\frac{-4x - 4}{(x+1)^4} = 0 \rightarrow x = -1$$

$$\int \frac{1}{x} dx = \ln x + C$$



Describe the nature of the curvature at $x=1/4$ for $f(x) = -x^3 + 2x^2 - x$

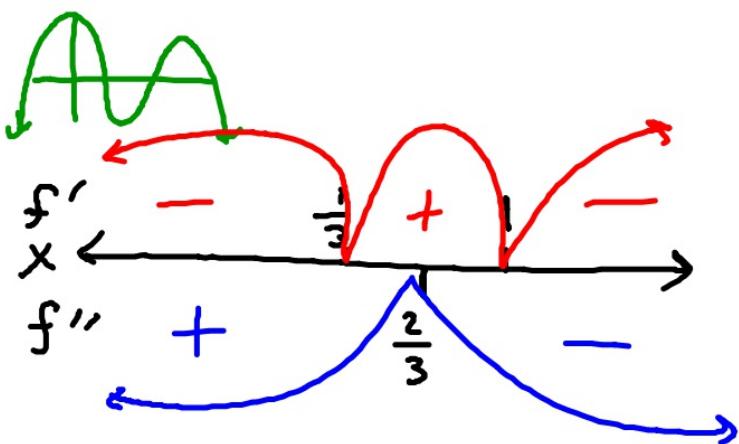


$$f'(x) = -3x^2 + 4x - 1 = 0$$

$$-1(3x^2 - 4x + 1) = 0$$

$$-1(3x - 1)(x - 1) = 0$$

$$\underbrace{x = \frac{1}{3}}_{C.N.} \quad \underbrace{x = 1}_{T.P.}$$



$$f''(x) = -6x + 4 \stackrel{C.N.}{=} 0 \rightarrow \underbrace{x = 2/3}_{T.P.}$$

Give any intervals where the function is decreasing and concave up
 $f(x) = x^3 - 2x^2$

$$f' = 3x^2 - 4x = 0$$

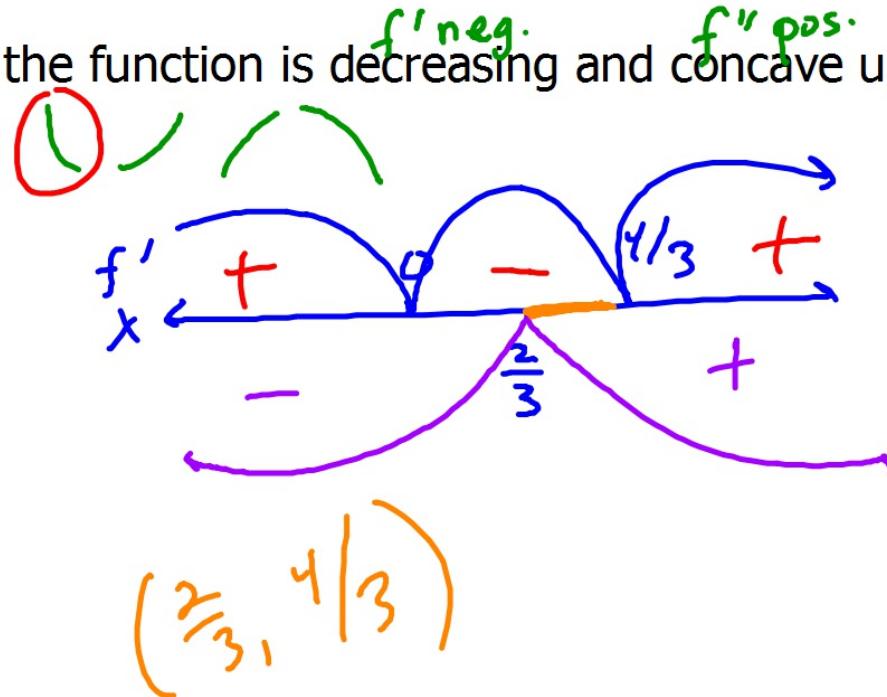
$$x(3x-4) = 0$$

$$\begin{cases} x=0 \\ x=\frac{4}{3} \end{cases}$$

SC.N.

$$f'' = 6x - 4 = 0$$

$$x = \frac{2}{3}$$



Practice Assessment