

H. Asymp.

1. $\lim_{x \rightarrow \infty} \frac{5x^2}{3x^2 + 100000x} =$

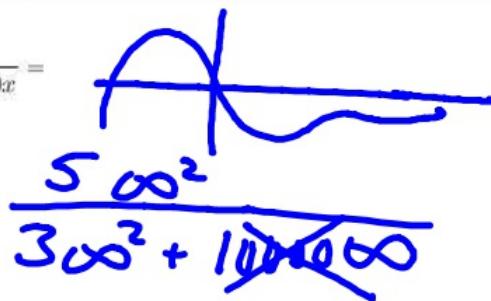
A) 0

B) 0.005

C) 1

D) 1.667

E) does not exist



2. Which of the following functions are not differentiable at $x = \frac{2}{3}$?

I. $f(x) = \sqrt[3]{x-2}$

II. $g(x) = |3x-2|$

III. $h(x) = |9x^2 - 4|$

A) I only

B) II only

C) I and II only

D) II and III only

E) I and III only

$$f = (x-2)^{1/3}$$

$$\frac{1}{3}(x-2)^{-2/3}$$

$$\frac{1}{3(x-2)^{2/3}}$$

$$9x^2 - 4 = 0$$

$$9x^2 = 4$$

$$x^2 = \frac{4}{9}$$

$$x = \pm \frac{2}{3}$$

3. If $y = (\ln x)^3$, then $dy/dx =$

A) $\frac{3}{x}(\ln x)^2$

$$y = (\ln x)^3$$

B) $3(\ln x)^2$

$$y' = 3(\ln x)^2 \cdot \frac{1}{x}$$

C) $3x(\ln x)^2 + (\ln x)^3$

D) $3(\ln x + 1)$

E) None of these

$$f' = 1 \quad g' = \cos(x)$$

4. If $F(x) = x \sin x$, then find $F'(3\pi/2)$.

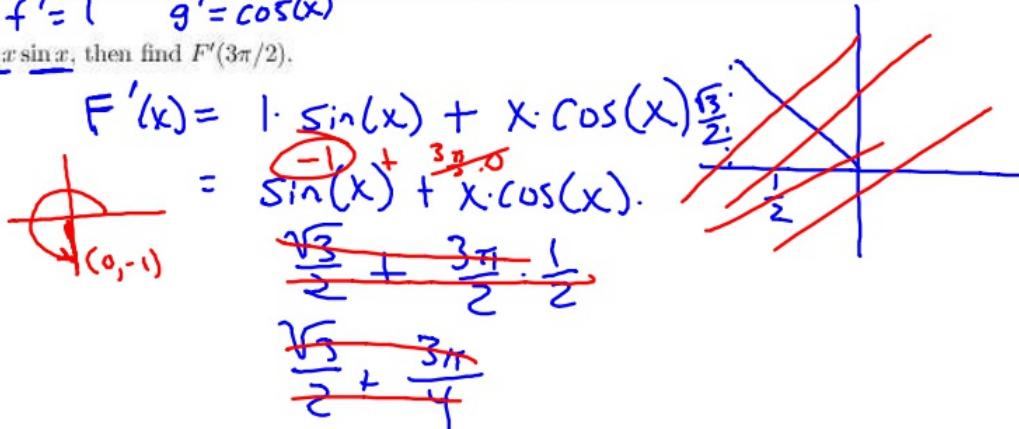
A) 0

B) 1

C) -1

D) $3\pi/2$

E) $-3\pi/2$



5. The approximate equation of the tangent line to $f(x) = \cos^2(3x)$ at $x = \pi/18$ is

A) $y = -2.598x + 1.203$

B) $y = 2.598x - 1.203$

C) $y = -2.598x + 0.575$

D) $y = 2.598x - 0.575$

E) None of these

$$f' = 2 \cos(3x) \cdot -\sin(3x) \cdot 3$$

$$- 6 \cos(3x) \sin(3x)$$

$$f'\left(\frac{\pi}{18}\right) = -2.598 \quad \left(\frac{\pi}{18}, 75\right)$$

$$y - 0.75 = -2.598\left(x - \frac{\pi}{18}\right)$$

Simplify to $y = mx + b$ form

Implicit Diff.

6. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at the point $(2, 1)$ is

A) $-\frac{3}{2}$

B) -1

C) $-\frac{5}{14}$

D) $-\frac{3}{14}$

E) 0

$(2, 1)$
x
y

$$3y^2 \frac{dy}{dx} \cdot x + y^3 \cdot 1 + 2y \frac{dy}{dx} \cdot x^2 + y^2 \cdot 2x = 0$$

$$3y^2 \frac{dy}{dx} \cdot x + 2y \frac{dy}{dx} \cdot x^2 = -2xy^2 - y^3$$

$$\frac{dy}{dx} (3xy^2 + 2x^2y) = -2xy^2 - y^3$$

$$\frac{dy}{dx} = \frac{-2xy^2 - y^3}{3xy^2 + 2x^2y} = \frac{-5}{14}$$

Differentiable

7. Which of the following functions has a derivative at $x = 0$?

- I. $y = \arcsin(x^2 - 1) - x$
- II. $y = x|x|$
- III. $y = \sqrt{x^4}$

- A) I only
- B) II only
- C) III only
- D) II and III only
- E) I, II, and III

8

Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

- (A) -0.701
- (B) -0.567
- (C) -0.391
- (D) -0.302
- (E) -0.258

$$\begin{aligned} f' &= 3e^{2x} \cdot 2 \\ f' &= 6e^{2x} = 18x^2 \end{aligned}$$

9.



$$f'(x) = 1$$

$$f(0.237) = 0.115$$

Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

- (A) $y = 8x - 5$
- (B) $y = x + 7$
- (C) $y = x + 0.763$
- (D) $y = x - 0.122$
- (E) $y = x - 2.146$

$$[f' = 4x^3 + 4x = 1]$$

$$x = 0.237$$

$$f'(0.237) = 1$$

m

10. A tangent line drawn to the graph of $y = \frac{4x}{1+x^3}$ at the point $(1, 2)$ forms a right triangle with the coordinate axes. The area of the triangle is

A) 3

B) 3.5

C) 4

D) 4.5

E) 5

$$f'(1) = -1$$

$$y - 2 = -1(x - 1)$$

$$y = -x + 1 + 2$$

$$y = -x + 3$$

$$A = \frac{1}{2}(3)(3) = \frac{1}{2} \cdot 9 = 4.5$$

11. The function

$$f(x) = \begin{cases} 4 - x^2 & x \leq 1 \\ mx + b & x > 1 \end{cases}$$

$$\begin{array}{l} m+b=3 \\ -x+b=3 \\ \hline b=5 \end{array}$$

is continuous and differentiable for all real numbers. What must be the values of m and b ?

A) $m = 2, b = 1$ B) $m = 2, b = 5$ C) $m = -2, b = 1$ D) $m = -2, b = 5$

E) None of these

$$f'(x) = \begin{cases} -2x & x \leq 1 \\ m & x > 1 \end{cases} \Rightarrow -2 \rightarrow m = -2$$

12. If $f(x) = -x^2 + x$, then which of the following expressions represents $f'(x)$?

A) $\lim_{h \rightarrow 0} \frac{(-x^2 + x + h) - (-x^2 + x)}{h}$

B) $\lim_{h \rightarrow x} \frac{(-x^2 + x + h) - (-x^2 + x)}{h}$

C) $\frac{[-(x+h)^2 + (x+h)] - (-x^2 + x)}{h}$

D) $\lim_{h \rightarrow 0} \frac{[-(x+h)^2 + (x+h)] - (-x^2 + x)}{h}$

E) None of these

13. All the functions below, except one, have the property that $f(x)$ is equal to its fourth derivative, $f^{(4)}(x)$. Which one does not have this property?

A) $f(x) = \sin x$

$$f \cdot e^{-x}$$

B) $f(x) = \cos x$

$$f' = -e^{-x}$$

C) $f(x) = -5e^x$

$$f'' = e^{-x}$$

D) $f(x) = e^{2x}$

$$f^3 = -e^{-x}$$

E) $f(x) = e^{-x}$

$$f^4 = e^{-x}$$

14. If $g(t) = \frac{\ln t}{e^t}$, then $g'(t) =$

$$\frac{1}{t} \cdot e^t - \ln t \cdot e^t$$

A) $\frac{1 - \ln t}{e^t}$

$$f' = \frac{1}{t}$$

$$(e^t)^2$$

B) $\frac{1 - t \ln t}{e^t}$

$$e^t \left(\frac{1}{t} - \ln t \right)$$

C) $\frac{t \ln t - 1}{te^t}$

$$(e^t)^{-1}$$

D) $\frac{1 - t \ln t}{te^t}$

$$\left(\frac{1}{t} - \ln t \right)$$

$$\frac{\left(\frac{1}{t} - \ln t \right)}{(e^t)} \cdot \frac{t}{t} = \frac{1 - t \ln t}{t e^t}$$

15. If $H(x) = x^3 - x^2 + \frac{1}{x}$, which of the following is $H''(2)$?

A) $\frac{31}{4}$

B) $\frac{39}{4}$

C) $\frac{79}{8}$

D) $\frac{81}{8}$

E) $\frac{41}{4}$

$$H''(2) = 6x - 2 + \frac{2}{x^3}$$

$$10 + \frac{1}{4}$$