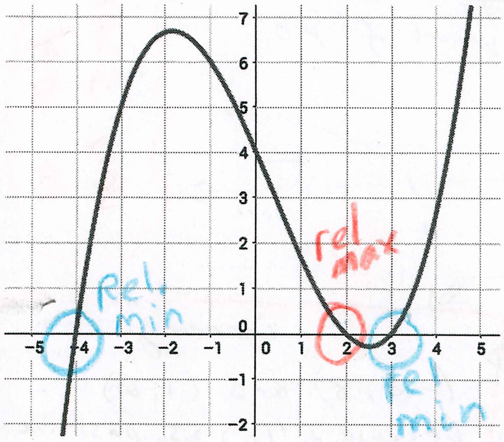
Sols

D-AD7 Practice Assessment

Given below is the graph of f' the first derivative of f. Use it to answer #1 and 2.



1. Over what interval(s) is f decreasing? Explain in detail.

means f' is negative ✓

$(-\infty, -4)$ and $(2,3)$. f' is negative here, so f has neg. slope.

f inc $\rightleftharpoons f' > 0$

f dec $\rightleftharpoons f' < 0$

2. Where, if anywhere, does f achieve a relative minimum? Justify your response.

f has relative minima @ $x=-4$ and $x=3$. because $f'=0$ there and changes from negative to positive.

f: dec → CN → increasing

f': neg → zero, undef → positive

Rel. max:

f: Inc → C.N. → dec.

f': pos → zero or undef. → neg.

D-AD8

3. Find the absolute extrema of $f(x) = 2x^3 - 6x - 2$ over the interval [-4,0].

① Critical Numbers

$f'(x) = 6x^2 - 6 = 0$

$6(x^2-1) = 0$

$6(x+1)(x-1) = 0$

$\underbrace{x=-1 \quad x=1}_{C.N.}$

② Try C.N. and Endpts.

$f(-1) = 2(-1)^3 - 6(-1) - 2$
$-2+6-2 = \underline{2}$ ← biggest

$f(1) = 2(1)^3 - 6(1) - 2$
$2-6-2 = \underline{-6}$

$f(-4) = 2(-4)^3 - 6(-4) - 2$
$-128 + 24 - 2 = \underline{-106}$ ← smallest

$f(0) = 2(0)^3 - 6(0) - 2 = \underline{-2}$

ABS. MAX $(-1, 2)$

ABS. MIN $(-4, -106)$

4. Find and classify all relative maxima and relative minima of $f(x) = -x^3 + 3x^2 + 2$. Justify your classifications.

↳ must occur @ critical numbers, but need to see sign change!

C.N.?

$f'(x) = -3x^2 + 6x = 0$

$-3x(x-2) = 0$

$-3x = 0 \qquad x-2=0$

$\underbrace{\underline{x=0} \qquad \underline{x=2}}_{C.N.}$

MIN MAX

f': $-$ $+$ $-$

x: -1000 0 1 2 1000

$f'(x) = -3x(x-2)$

- -1000 → $(+)(-) = -$
- 1 → $(-)(-) = +$
- 1000 → $(-)(+) = -$

① Plot C.N. on number line

② Determine sign in intervals by plugging test values into f'

$x=0$ is a rel. min. because $f'(0)=0$ and sign change neg → pos. $x=2$ is a rel. max because $f'(2)=0$ and sign change pos → neg.

D-AD9

5. For what interval(s) is the function $f(x) = x^3 + 3x^2 - 9x + 7$ increasing? Justify your answer.

↳ means $f' > 0$

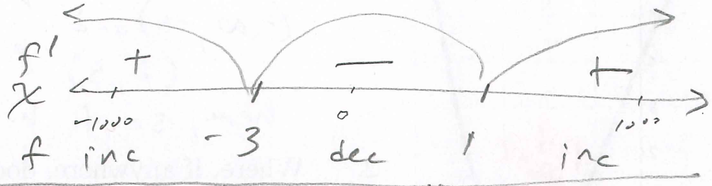
Find C.N.

$f'(x) = 3x^2 + 6x - 9 = 0$

$3(x^2 + 2x - 3) = 0$

$3(x+3)(x-1) = 0$

$\underbrace{x = -3 \quad x = 1}_{\text{C.N.}}$



$f'(-1000) = 3(-)(-) = +$

$f'(0) = 3(3)(-1) = -$

$f'(1000) = 3(+)(+) = +$

$\boxed{f \text{ is increasing from } (-\infty, 3) \text{ and } (1, \infty) \text{ because } f'(x) \text{ is positive there.}}$

Very Similar to #4

D-CD8

6. Find the value of c guaranteed to exist by the Mean Value Theorem for $f(x) = x^3 - 2x^2$ over $[0,2]$.

8. Use a tangent line to approximate $\sqrt[3]{122}$

- fixed point: $(125, 5)$
  (x_1, y_1)
- function: $f(x) = \sqrt[3]{x} = x^{1/3}$

$y - 3 = m(x - 125)$

$m?$ $f'(125)$ $\longrightarrow$ $f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$= \frac{1}{3(\sqrt[3]{x})^2}$

$f'(125) = \frac{1}{3(\sqrt[3]{125})^2}$

$\frac{1}{3(5)^2} = \frac{1}{3 \cdot 25} = \frac{1}{75}$

$y - 5 = \frac{1}{75}(x - 125)$

$x = 122$

$y - 5 = \frac{1}{75}(122 - 125)$

$y - 5 = \frac{1}{75}(-3)$

$y - 5 = -\frac{3}{75}$

$y = 5 - \frac{3}{75}$

$\boxed{y = 4\frac{72}{75}}$

D-AD5

7. Find $\frac{dy}{dx}|_{(1,2)}$ for $\underline{xy^2} + \underline{2xy} = 8$

f, g

Product $\quad$ product

$f: x \quad g: y^2$

$f': 1 \quad g': 2yy'$

$f'g + fg'$

$f: 2x \quad g: y$

$f': 2 \quad g': y'$

$f'g + fg'$

$1y + x \cdot 2yy' + 2y + 2xy' = 0$

$\underline{y} + 2xyy' + \underline{2y} + 2xy' = 0$

$2xyy' + 2xy' = -3y$

$y'(2xy + 2x) = -3y$

$y' = \frac{-3y}{2xy + 2x} \xrightarrow{\text{plug in } (1,2)} \frac{-3(2)}{2(1)(2) + 2(1)} = \frac{-6}{4+2} = \boxed{-1}$

8. Show that $\frac{dy}{dx} = -\frac{2x+6xy}{3x^2+3y^2}$ for $x^2 + 3x^2y + y^3 = 12$

f / g

$\underline{2x} + \underline{6xy} + 3x^2y' + 3y^2y' = 0$

$3x^2y' + 3y^2y' = -2x - 6xy$

$y'(3x^2 + 3y^2) = -2x - 6xy$

$y' = \frac{-2x - 6xy}{3x^2 + 3y^2}$

$y' = -\frac{2x - 6xy}{3x^2 + 3y^2}$

D-CD2

9. A water tank has a hole in it and is being slowly drained. Suppose the volume of water V, in liters, in the tank at any time t, in seconds, is given by the twice differentiable function $\boldsymbol{W(t)}$. At time t=0, there are 20 liters in the tank.

liters → $w'(t) = \frac{lit}{sec}$ rate of change

   a. Is $\boldsymbol{W'(t)}$ always positive or negative? Explain.

Negative. The volume of water in the tank decreases as it's drained.

RATE/2

   b. Explain the meaning of $\boldsymbol{W'(b) = a}$ using correct units.

$\frac{dW}{dt} = \frac{\ell}{sec}$

time rate of water loss

At b seconds, the volume of the water is changing at a rate of a liters per second.

10. The country of Mozambique's population can be modeled by the twice differentiable function M(t) where M measures the population in millions of people and t is measured in years after 1950. Explain the meaning of $\frac{d^2M}{dt^2}|_{t=50} = 0.25$ using correct units. → 250,000

2nd derivative: rate of change of the rate of change

→ t = 50 → 50 yrs after 1950 → yr: 2000

→ $\frac{dM}{dt} = \frac{\text{millions of people}}{\text{year}}$ population rate

first derivative

$\frac{d^2M}{dt^2} \leftarrow \frac{\frac{\text{millions of people}}{\text{year}}}{\text{year}}$ or $\frac{\text{millions of people}}{\text{year}^2}$ change in population rate.

In 2000, Mozambique's population rate (2nd deriv) is changing at a rate of 250,000 people per year per year.
(or 0.25 million people per year per year).