

Good afternoon: do the warm up in your notebook

Evaluate each limit

$$\lim_{t \rightarrow 0} \frac{\sin 4t}{3t} \quad \frac{0}{0} \text{ ??}$$

$$\lim_{t \rightarrow 0} \frac{4 \cos 4t}{3} = \frac{4}{3}$$

$$\lim_{t \rightarrow 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} = \frac{0}{0}$$

$$\lim_{t \rightarrow 1} \frac{20t^3 - 8t}{-1 - 27t^2} = \frac{12}{-28} \rightarrow \frac{3}{7}$$

Reminders

Next assess: 11/30

505 a5 c8 d2 e6 f -1

521

- a. $(3x^2y - y^2)/(2xy - x^3)$
- b. $y=3$ and $y+2=2(x-1)$
- c. $(-24)^{1/5}$

522

- a. use implicit diff
- b. $y-1=1/4(x+2)$
- c. $(2, -1)$ $(-4, -1)$
- d. No, h.t. at $x=-1$ and $(-1, 0)$
does not satisfy given equation

523 a use imp. diff

b. $(0, \sqrt{2}), (0, -\sqrt{2})$

c. h.t. means $y=0$, plug in to original
and get false result

$$\frac{dy}{dx} = \frac{3x^2y - 4y^2}{2xy - x^3}$$
$$(1, 3) \left| \begin{array}{l} m=0 \\ m=2 \end{array} \right.$$

~~$x=1$~~ $xy^2 - xy = 6$

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y=3 \quad y=-2$

$$596. 3\csc(2x)(1-2x\cot(2x))$$

$$598. \frac{-5\csc^2 5x}{2\sqrt{\cot 5x}}$$

$$600. (\sin(x)-x*\ln(x)*\cos(x))/(x*\sin^2(x))$$

$$602. e^{\sin x} \cos x \quad 534.$$

$$604. \frac{2}{\ln 3} \cot 2x$$

$$606. e^{3x}(\sec^2 x + 3 \tan x) \quad r^3 - 243$$

$$535. 6\sqrt{3} \text{ ft} \quad 536. 2\sqrt{3} \text{ u}$$

$$608. \frac{1}{2}xe^{x^2/4}$$

$$610. e^{\tan x}(1 + x \sec^2 x)$$

$$525: 216u^3, 216u^2$$

$$526: 60u^3, 94u^2$$

$$527. 13 \text{ u}$$

$$528. 4\sqrt{2} \text{ u}$$

$$529. 5\sqrt{5} \text{ u}$$

$$530. BC=12.5 AC=13.463$$

$$531. 16\sqrt{3} \text{ u}^2$$

$$532. 50\pi \text{ u}^2$$

$$533. 15 \text{ cu ft}$$

$$534. 6.240 [(243)^{1/3}]$$

Using implicit differentiation to find a second derivative (notes)

$$\frac{d}{dx}(y^2 - xy) = 8 \quad \text{Find } y'$$

$$2yy' - [1y + x \cdot y'] = 0$$

$$f: x \quad g: y$$

$$f': 1 \quad g': y'$$

$$2yy' - y - xy' = 0$$

$$2yy' - xy' = y$$

$$y'(2y - x) = y$$

$$\frac{d}{dx}(y) = \left[\frac{y}{2y-x} \right] \frac{d}{dx} x \quad f: y \quad g: 2y-x$$

$$y'' = \frac{y' \cdot (2y-x) - y \cdot (2y'-1)}{(2y-x)^2} \quad f': y' \quad g': 2y' - 1$$

$$= \frac{2y'y - xy' - 2yy' + y}{(2y-x)^2}$$

See a bar

$$y'' = \frac{-x(y') + y}{(2y-x)^2} \rightarrow \boxed{\frac{-x \cdot \frac{y}{2y-x} + y}{(2y-x)^2}} \quad \text{good enough for now.}$$

to simplify, multiply by
'compound denominator'
 $2y-x$ over itself

$$\frac{2y^2 - 2xy}{(2y-x)^3}$$

$$\left(\frac{-xy}{(2y-x)^2} + \frac{y}{(2y-x)^2} \right) \cdot \frac{1}{(2y-x)}$$

$$\left\langle \frac{-xy + 2y^2 - xy}{(2y-x)^3} \right\rangle$$

Find y''

$$2x^3 - 3y^2 = 8$$

$$6x^2 - 6yy' = 0$$

$$\frac{-6yy'}{-6y} = \frac{-6x^2}{-6y}$$

$$\frac{1}{2x} \left[y' = \frac{x^2}{y} \right]$$

$$\begin{array}{ll} f: x^2 & g: y \\ f': 2x & g': y' \end{array}$$

$$y'' = \frac{d^2y}{dx^2} = \frac{2xy - x^2 \cdot y'}{y^2} \rightarrow \frac{\left(2xy - x^2 \cdot \frac{x^2}{y} \right)}{y^2} \cdot \frac{y}{y}$$

$$y'' = \frac{2xy^2 - x^4}{y^3}$$

$$\frac{2xy^2 - \frac{x^4}{y} \cdot y}{y^3}$$

Derivatives of Inverse Functions (notes)

What are inverse functions?

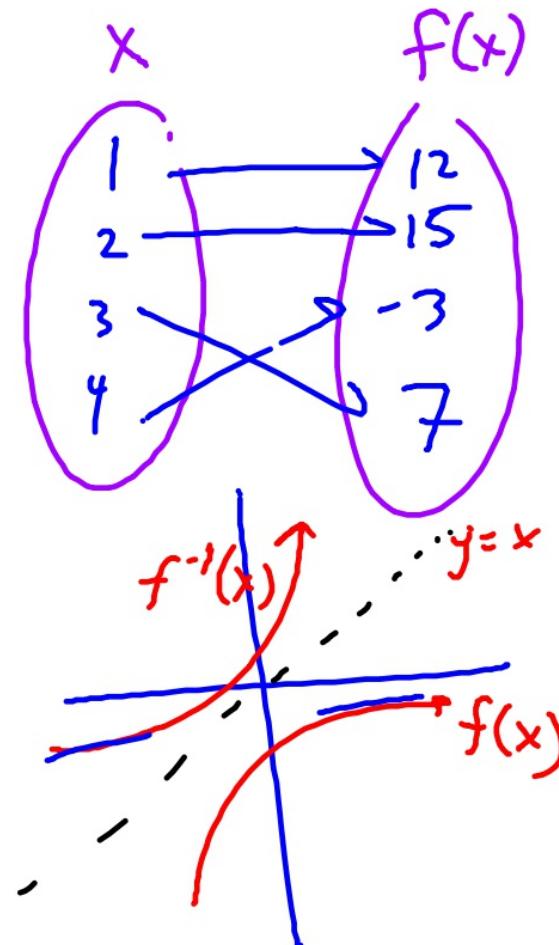
Inverse, not $\frac{1}{y}$

Find y^{-1} if $y=2x-6$

$$x = 2y - 6$$

$$x+6 = 2y$$

$$\boxed{\frac{1}{2}x + 3 = y^{-1}}$$



A property of inverse functions:

$$f(g(x)) = x$$

$$f: 2x - 6$$

$$g: \frac{1}{2}x + 3$$

ex: square root and²

$$\begin{array}{rcl} y & & \\ 2\left(\frac{1}{2}x + 3\right) - 6 & = & y \\ x + 6 - 6 & = & x \\ x & & \end{array}$$

$$\begin{array}{rcl} \sqrt{x^2} & = & x \\ \sqrt{6^2} & = & 6 \end{array}$$

If f and g are inverse functions, then $f(g(x)) = x$

$$\frac{d}{dx} f(g(x)) = \frac{d}{dx} x$$

$$\underline{f'(g(x)) \cdot g'(x)} = \underline{1}$$

$$f'(g(x)) \quad f'(g(x))$$

$$g'(x) = \frac{1}{f'(g(x))} *$$

ADD TO BOOKLET
(Derivative of Inverses)

$$g'(x) = \frac{1}{f'(g(x))}$$

If $f(x) = x^5 + 2x - 1$ then find $f^{-1}'(2)$.

Let $g(x) = f^{-1}(x)$

①

guess
rank

$$\begin{cases} x^5 + 2x - 1 = 2 \\ 1^5 + 2(1) - 1 \end{cases}$$

$$1 + 2 - 1$$

$$2 \leq 2$$

$$* f:(1, 2)$$

$$g:(2, 1)$$

① Set $f(x) = 2$

② Take f'

$$f'(x) = 5x^4 + 2$$

$$③ g'(x) = \frac{1}{f'(g(x))}$$

$$g'(2) = \frac{1}{f'(g(2))}$$

$$g'(2) = \frac{1}{f'(1)} = \frac{1}{5 \cdot 1^4 + 2} = \frac{1}{7}$$

If f and g are inverses and $f(x) = 4x^5 + 3x^3$, find $g'(7)$

$$4x^5 + 3x^3 = 7$$

uh... try 1?

$$4 \cdot 1^5 + 3 \cdot 1^3 = 7$$

$$4+3=7 \checkmark$$

$$f:(1, 7)$$

$$g:(7, 1)$$

$$f': 20x^4 + 9x^2$$

$$g'(7) = \frac{1}{f'(g(7))}$$

$$g'(7) = \frac{1}{f'(1)}$$

$$g'(7) = \frac{1}{29}$$

The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

$$f: 3x^2 + 4$$

$$g: (2, 0)$$

$$g'(2) = \frac{1}{f'(g(2))}$$
$$g'(2) = \frac{1}{f'(0)} = \boxed{\frac{1}{f}}$$

Have a Happy Thanksgiving!



Finish L'hopital's rule hw p 564

11/30 assessment:

D-AD0: L'hopital

D-AD6: Inverse Deriv.

D-CD5: H/V. Tangents.

