Let's put the mean value theorem on pause

- intuitive notion: for differentiable functions, a function must take on its average slope at at least one point in an interval.

Will have more time on Friday

Outline before Winter Break:

11/30 Today: Extrema, Critical Numbers

12/2 Wed: First and Second derivative tests

12/4 Fri: Assessment[1] (AD789, 10, 11); Mean Value Theorem

Thu after school review session

12/7 Mon: Derivatves MC test (45 min, calculators)

Bingo/Review due

12/9 Wed: Related Rates: using implicit diff.

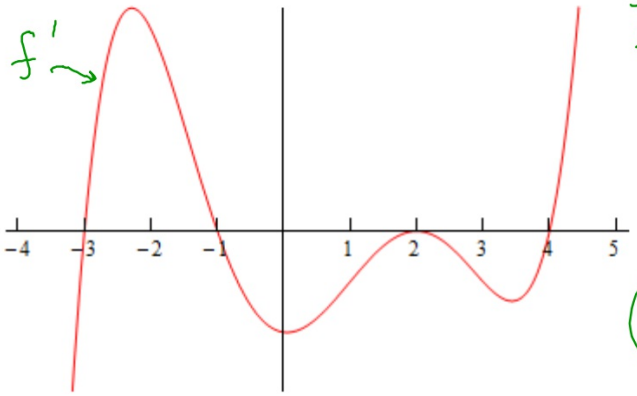
12/11 Fri: Assessment[1] (CD 8AD 12, 13), Optimization

12/14 Mon: Assessment[1] (AD 14, 15, 16)

12/16 Wed: Linearization, and Assessment (AD 18)

[1] will do practice tests for each

Here is a graph of f '(x), the derivative of a function f(x).
Where is f (not shown) increasing? Where is it decreasing? Justify your answer.



Graph of the derivative of f

f increasing $\Rightarrow$ f' is positive
$(-3,-1)$ and $(4,\infty)$

f decreasing $\Rightarrow$ f' negative
$(-\infty,-3)$ $(-1,2)$ $(2,4)$.

A function f is increasing when....

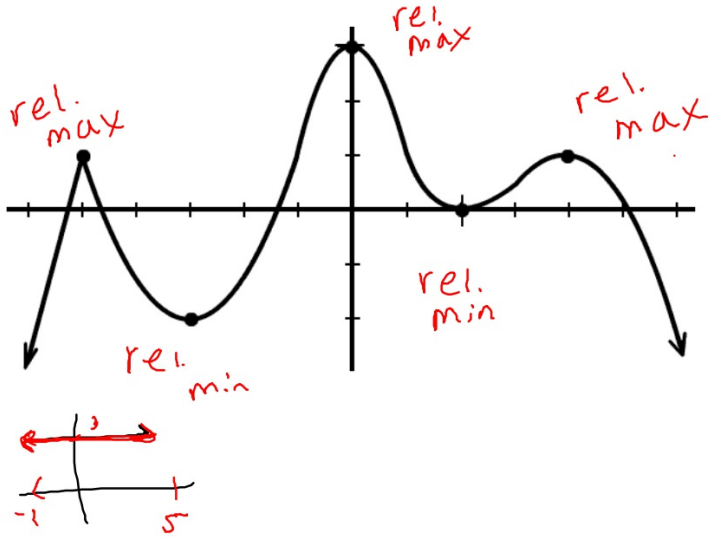
A function f is decreasing when....

A preview...

Two different types of increase:

Two different types of decrease:

Where are the relative extrema of this function?



Relative Maximum:
- slope at point? 0 or undef.
- slope before point? pos
- slope after point? neg.

Relative Minimum:
- slope at point? 0 or undef
- slope before point? neg
- slope after point? pos

[redacted]

[redacted]

How to find relative extrema: First Derivative Test

- Find critical numbers of the function.
- Use the "interval method"/number line to find a sign change
- Use common sense/intuition to classify which C.N.'s are maxima and minima

(Also called the "First Derivative Test")

Example: Find and classify all relative extrema of the function.Justify your answer.

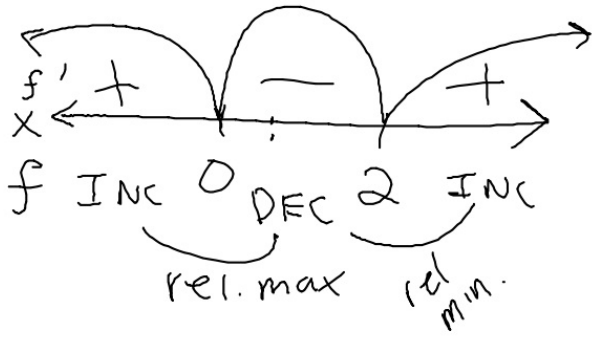
$f(x) = x^3-3x^2+5$

$f'(x) = 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x=0 \quad x=2$

C.N.

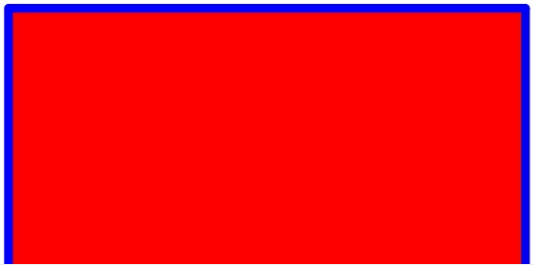


f has a rel. max @ x=0
b/c. f' changed from pos → neg.

f has a rel. min @ x=2
b/c ... f' changes sign from neg → pos.

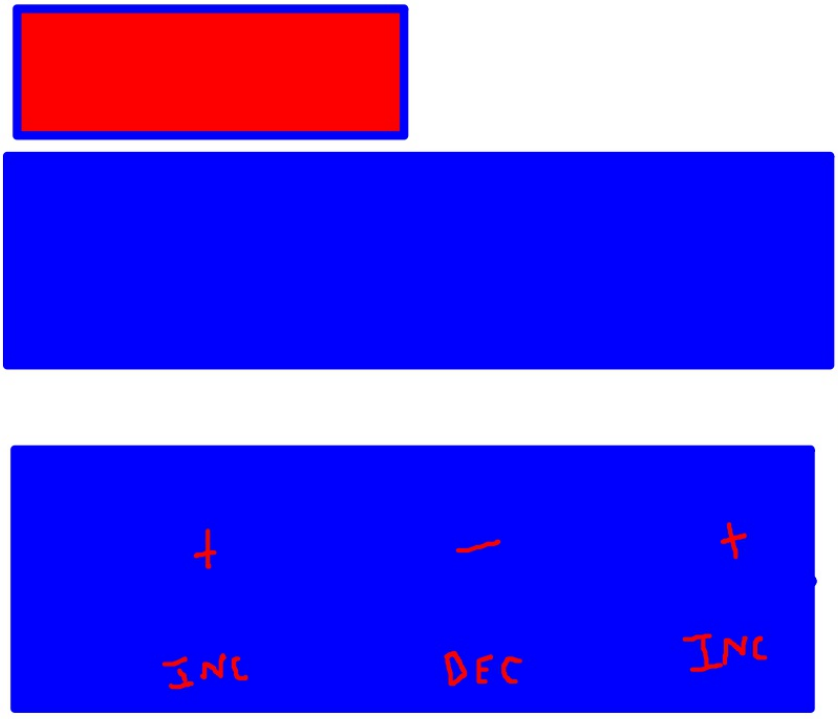
Using a Number Line
- place C.N. on there
- Bunny Hops
- Test values between hops
  (plug into f '(x))

A Number Line Is NOT sufficient Justification on AP test. Need to use English

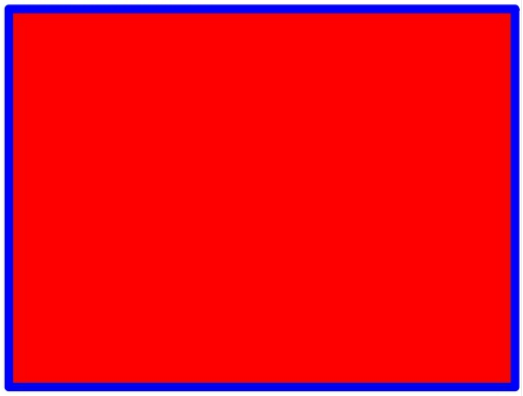


Your turn: Find and classify all relative extrema. Justify.

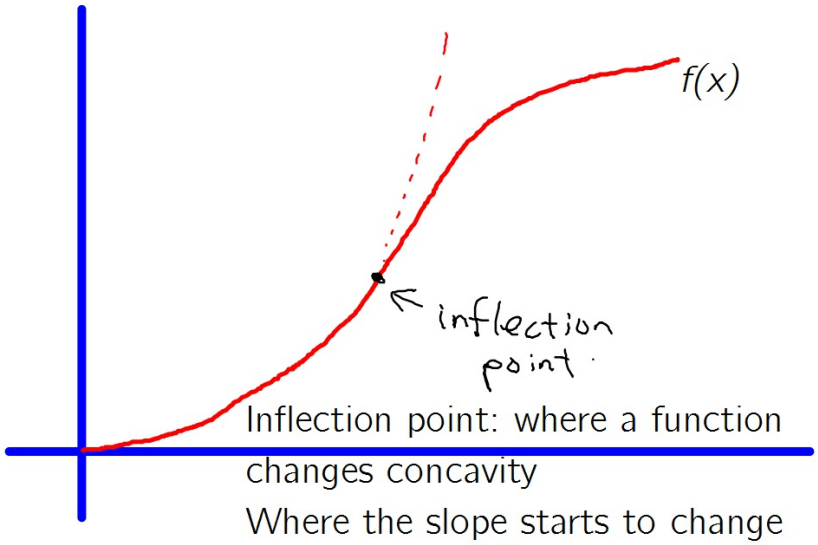
$f(x) = x^3+6x^2+9x+6$



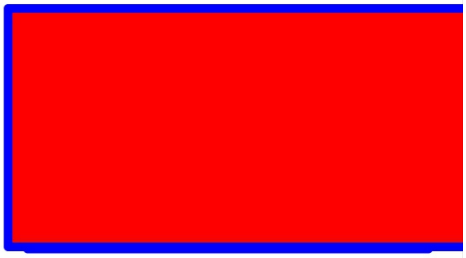
Justification:



Consider the following differentiable curve: Concavity



What might this model?

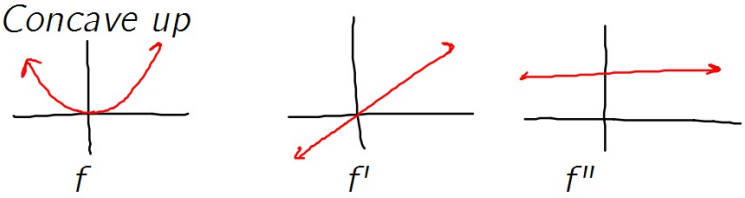


Inflection point: where a function changes concavity

Where the slope starts to change (second derivative)

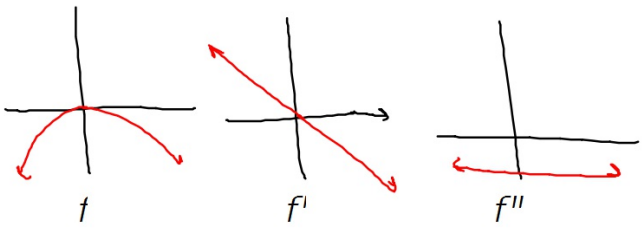
Concavity:

Concave up



f con. up,
f'' is positive

Concave down

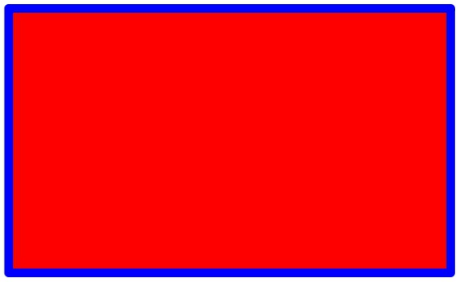


f is conc. Down,
f'' is negative

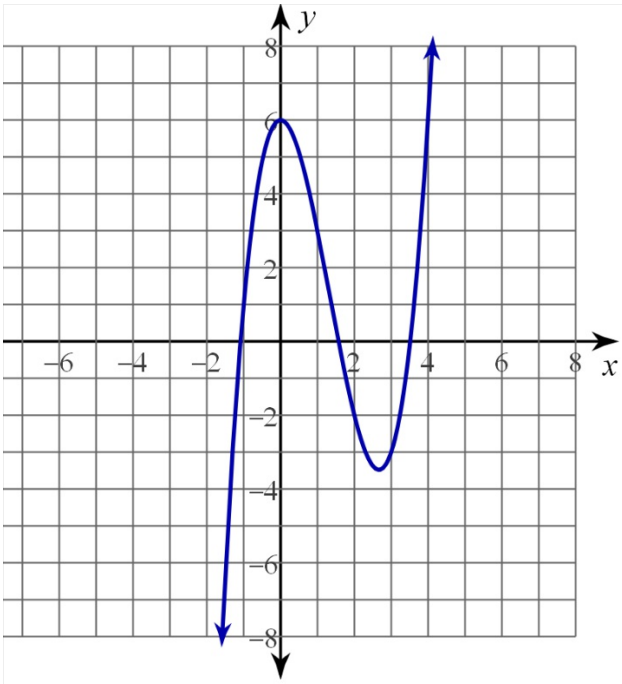
The sig
derivati
concave

Terrace Points *(similar to critical points)*
where f " (x) = 0

Inflection Points *(similar to relative extremes)*
where f"(x) changes sign
graphically: a change in concavity



Four Types of Curvature



Example:

Here is the first derivative of f(x). Over what intervals is f concave up and concave down?

Find the location of all inflection points for $f(x) = x^3-x^2-1$

Just like finding relative extrema.. except using f"