Ungraded Practice Problems Because Mr. Mohyuddin is at the Dentist (UPPBMMIATD)

1. Use the definition of continuity to show that *f(x)* is continuous at $x = 2$.

$$f(x) = \begin{cases} 5x - 3x^2, x \le 2 \\ 3x - 8, \quad x > 2 \end{cases}$$

$$\lim_{x \to 2^-} f(x) = f(2) = \lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^-} [5x - 3x^2] = 5(2) - 3(2)^2 = \lim_{x \to 2^+} [3x - 8]$$

$$5(2) - 3(2)^2 = 10 - 3(4) = 3(2) - 8$$

$$10 - 3(4) = 10 - 12 = 6 - 8$$

$$10 - 12 = -2$$

$$-2 = -2 = -2$$ ⟹ continuity confirmed!

2. Find the values of *a* and *b* to make *f(x)* continuous everywhere. $f(x) = \begin{cases} 0.2x^3 - 2 & x < 0 \\ ax + b & 0 \le x < 4 \\ 10 - \frac{1}{2}x & x \ge 4 \end{cases}$

Need only check continuity @ $x = 0$ & $x = 4$ b/c all 3 cases are polynomial or linear

Cont. @ $x = 0$?

$$\lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$$

$$\lim_{x \to 0^-} [0.2x^3 - 2] = a(0) + b = \lim_{x \to 0^+} [ax + b]$$

$$-2 = b = b$$

So, $b = -2$

Cont. @ $x = 4$?

$$\lim_{x \to 4^-} f(x) = f(4) = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} [ax + b] = a(4) + b = \lim_{x \to 4^+} [10 - \tfrac{1}{2}x]$$

$$4a + b = 4a + b = 10 - \tfrac{1}{2}(4)$$

$$4a + b = 10 - 2$$

$$4a + b = 8$$

Sub in $b = -2$

$$4a - 2 = 8$$

$$4a = 10$$

$a = \frac{10}{4}$ and $b = -2$

3. Find and classify any discontinuities of $f(x) = \frac{3x+9}{x^2-4x-21}$

$$f(x) = \frac{3(x+3)}{(x+3)(x-7)} = \frac{3}{x-7}$$

$x = -3$ ... removable ?!

↳ Need to show $\lim_{x \to -3} f(x)$ exists.

$$\lim_{x \to -3} \frac{3}{x-7} = \frac{3}{-3-7} = -\frac{3}{10}$$

So $x = -3$ is removable discontinuity.

$x = 7$ ... infinite ?!

↳ Need to take one-sided limit to yield + or − ∞.

$$\lim_{x \to 7^+} \frac{3}{x-7} = \frac{3}{7^+-7} = \frac{3}{0^+} = \infty$$

So $x = 7$ is an infinite discontinuity.

4. Show that $\lim_{x \to 8} \frac{|8-x|}{24-3x}$ does not exist.

→ $\lim_{x \to 8^-} f(x) \neq \lim_{x \to 8^+} f(x)$

Let $f(x) = \frac{|8-x|}{24-3x} = \frac{|8-x|}{3(8-x)}$ → rewrite as piecewise

↙ Set denom = 0, solve

$3(8-x) = 0 \Rightarrow x = 8$ "handoff point"

$$f(x) = \begin{cases} \frac{8-x}{3(8-x)} & x < 8 \\ \frac{-(8-x)}{3(8-x)} & x > 8 \end{cases}$$

Simplify ↓

$$f(x) = \begin{cases} \frac{1}{3}, & x < 8 \\ -\frac{1}{3}, & x > 8 \end{cases}$$

$\Rightarrow \lim_{x \to 8^-} f(x) = \lim_{x \to 8^-} \frac{1}{3} = \frac{1}{3}$

$\neq$

$\lim_{x \to 8^+} f(x) = \lim_{x \to 8^+} -\frac{1}{3} = -\frac{1}{3}$

So $\lim_{x \to 8} f(x)$ d.n.e.

q.e.d. ∎