Good afternoon: warm up

Use the definition of continuity at a point to show that f(x) is continuous at x=3.

$$f(x) = \begin{cases} 3x-1, & x<3 \\ 17-x^2, & x=3 \\ 5+3\cos(x-3), & x>3 \end{cases}$$

$$\lim_{x\to 3^-} \overset{3x-1}{\cancel{f(x)}} = \overset{17-x^2}{\cancel{f(3)}} = \lim_{x\to 3^+} \overset{5+3\cos(x-3)}{\cancel{f(x)}}$$

$$8 = 8 = 8$$

Definition of Continuity at a Point

$\Leftrightarrow$ iff

A function *f(x)* is continuous at a point x=a if and only if

$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

"left road" "bridge" "right road"

Alt: $\lim_{x \to a} f(x) = f(a)$

Continuity on an Interval

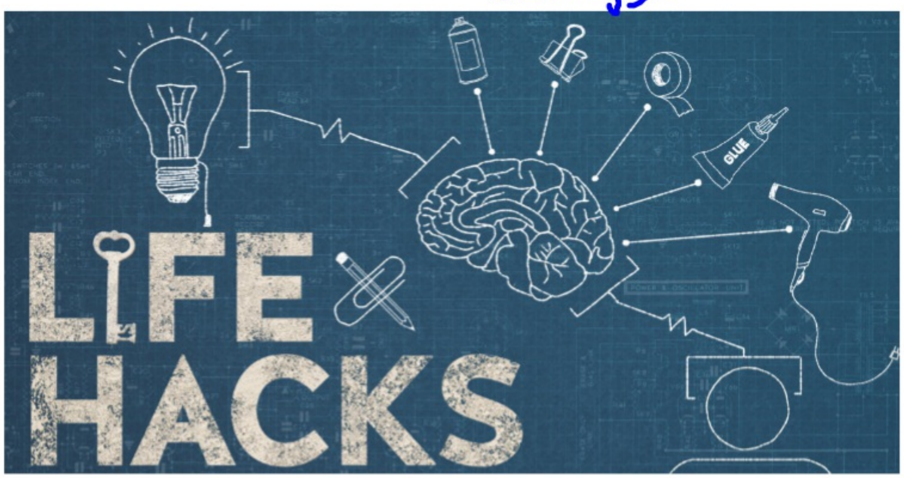
A function is continuous on an interval if it is continuous at every point *in* the interval

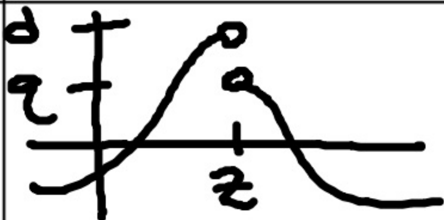
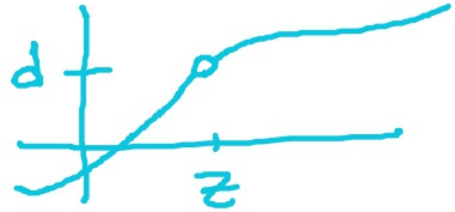
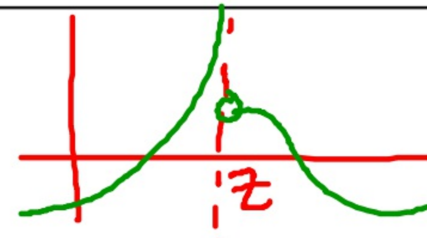
A function is continuous everywhere except where it is not

Save time:

- polynomial functions are always continuous everywhere
- piecewise functions often yield "jump" discontinuities
- sin(x) and cos(x) are always continuous everywhere
- rational functions often have v.a. or holes ("removable") discontinuities

(Inf.)



	Jump	Removable	Infinite
Graphically			
Algebraically	$\lim_{x\to z^-} f(x) = d$ $\lim_{x\to z^+} f(x) = q$ $d \neq q;\ d, q \in \mathbb{R}$	$\lim_{x\to z} f(x) = d$ • $f(z) =$ undef. $\varnothing$ • $f(z) \neq d$	$\lim_{x\to z^\pm} f(x) = \pm\infty$
Notes	→ usually piecewise	→ factor & cancel → limit exists	→ Identical to V.A.

Discuss the continuity of f(x). Classify any discontinuities and justify with limits.

$$f(x)= \frac{x+1}{2x^2-6x-8}$$

$$\frac{x+1}{2(x^2-3x-4)}$$

$$\cancel{\frac{x+1}{2(x+1)(x-4)}}$$

$$\frac{1}{2(x-4)}$$

$x=-1$ is remov.

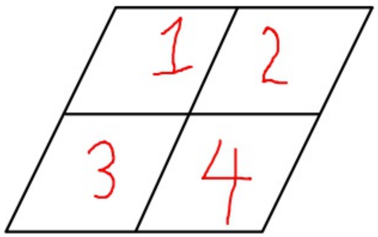
b/c $\lim\limits_{x\to -1} f(x) = \frac{1}{2(-1-4)} = -\frac{1}{10}$

$x=4$ inf. disc.

b/c $\lim\limits_{x\to 4^+} f(x) = \frac{1}{2(4^+-4)} = \frac{1}{2(0^+)}$

$= \frac{1}{0^+} = \infty.$

front of room



back of room

Each person does their own
problem using private time

Then person 1 shares findings
with group, others listen, write, ask q's

Then person 2 shares, etc.

① $f(x) = \begin{cases} x + \frac{1}{2}, & x < \frac{1}{2} \\ 0, & x \geq 1/2 \end{cases}$

Everywhere cont. .. maybe not @ $x = 1/2$?

$\lim\limits_{x \to 1/2^-} (x + 1/2) = \underline{1}$ $\qquad f(1/2) = 0$

$\lim\limits_{x \to 1/2^+} 0 = 0$ $\qquad$ Jump $0 \neq 1$

$\boxed{\text{f(x) cont. everywhere except } x = 1/2 \text{ (Jump)}}$

② $f(x) = -\frac{x^2 - 3x}{x}$

$$-\frac{\cancel{x}(x-3)}{\cancel{x}}$$

$$-(x-3)$$

$\lim_{x \to 0} -(x-3) = 3$

$f(0) = \emptyset$

! rem @ $x = 0$?

f is C.E. except $x = 0$ (rem.)

(3) $f(x) = \dfrac{x^2}{3x-9}$

$$\frac{x^2}{3(x-3)}$$

$x = 3$? Inf.?

$$\lim_{x \to 3^+} \frac{x^2}{3(x-3)}$$

$$\frac{9^+}{3(0^+)} = \frac{9^+}{0^+} = \infty$$

f is C.E. except $x=3$ (inf.)

$$f(x) = \frac{x-2}{x^2-x-2}$$

$$\frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

!

$x=2$ … rem?

$$\frac{1}{x+1}$$ ← $x=-1$ … inf ?

$$\lim_{x\to 2} \frac{1}{x+1} = \frac{1}{3} \checkmark$$

$$\lim_{x\to -1^-} \frac{1}{x+1} = \frac{1}{-1^-+1} = \frac{1}{0^-} = -\infty \checkmark$$

$-1.01+1 = -.01$

f is C.E.
except $x=2$ (rem), $x=-1$ (inf.)

Discuss the continuity of the function. Classify any discontinuities and justify classifications with limits.

1 $f(x) = \begin{cases} x + \frac{1}{2}, & x < \frac{1}{2} \\ 0, & x \geq \frac{1}{2} \end{cases}$

2 $f(x) = -\frac{x^2 - 3x}{x}$

3 $f(x) = \frac{x^2}{3x - 9}$

4 $f(x) = \frac{x - 2}{x^2 - x - 2}$

Making a function continuous

Find the value of *a* so that *f(x)* is continuous everywhere

$$f(x) = \begin{cases} 2x + 5 & x \leq 1 \\ ax + 2 & x > 1 \end{cases}$$

$$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$$

$$2(1) + 5 = 7$$

$$7 = 7 = a \cdot 1 + 2$$

$$7 = a + 2 \rightarrow a = 5$$

Find the values of *a* and *b* such that g(x) is continuous

http://bit.ly/detcont3

$$g(x) = \begin{cases} -2x^2 + 3, & x < 0 \\ ax + b, & 0 \le x \le 1 \\ 9x & x > 1 \end{cases}$$

$x=0$

$$\lim_{x\to 0^-} g(x) = g(0) = \lim_{x\to 0^+} g(x)$$

$$3 = b = b$$

$$a = 6$$

$x=1$

$$\lim_{x\to 1^-} g(x) = g(1) = \lim_{x\to 1^+} g(x)$$

$$a+b = a+b = 9$$

$$\underline{a+b=9}$$

Practice Assessment

F-L1c: rationalization + special trig limits
F-L1b: Infinite Limits: solve limits at/yielding infinity
F-B1: Asymptotes as Limits: find asymptotes and justify with limits
F-C1: Definition of continuity (like today's warm up)
F-C2: Making a function continuous
F-C3: Classifying Discontinuities (like hw, group work)