Good afternoon: warm up

Find any asymptotes for $f(x)=\frac{2x^2+14x+24}{x^2+x-6}$ Use limits to justify.

Not a v.a.!!

$$f(x)=\frac{2(x^2+7x+12)}{(x+3)(x-2)}=\frac{2\cancel{(x+3)}(x+4)}{\cancel{(x+3)}(x-2)}$$

$$\frac{2(x+4)}{x-2}$$

v.a. $x=2$?
$x=-3$?

$$\lim_{x\to 2^+}\frac{2(x+4)}{x-2}=\frac{12}{2^+-2}=\frac{12}{0^+}$$

$x=2$ is a v.a. $=\infty$

H.A.

$$\lim_{x\to\infty}\frac{2x^2+14x+24}{x^2+x-6}\quad \frac{2\infty^2}{\infty^2}=2$$

$y=2$ is a h.a.

$$f(x)=\frac{3x^3}{-5x^3}\ldots$$

Next assessment:
Thurs 9/6

HW Questions?

⑦ $\lim_{x \to 2} \frac{x-2}{\sqrt{x+7}-3} \cdot \frac{\sqrt{x+7}+3}{\sqrt{x+7}+3}$

$\frac{x+5}{\sqrt{4x^2+6}}$

⑰ $\lim_{x \to \infty} \sqrt[3]{\frac{4x^2+1}{x^2+2}}$

$\lim_{x \to \infty} \frac{\sqrt[3]{4x^2+1}}{\sqrt[3]{x^2+2}}$ $\frac{\sqrt[3]{4\infty^2}}{\sqrt[3]{\infty^2}} = \frac{\sqrt[3]{4} \cdot \cancel{\sqrt[3]{\infty^2}}}{\cancel{\sqrt[3]{\infty^2}}}$

Where we left off....

$$\lim_{x\to\infty} \frac{2^x}{3x^2} \qquad \frac{2^{\infty}}{3\infty^2} \qquad \frac{+\;+\;+}{+\;+\;+} = \infty$$

desmos

$$\lim_{x\to\infty} \frac{\ln\left(x^5\right)}{x^{0.02}} = 0$$

$$\frac{\text{grows slow}}{\text{grows fast}}$$

MYTH: Calculus is all about taking limits of more and more complicated functions.

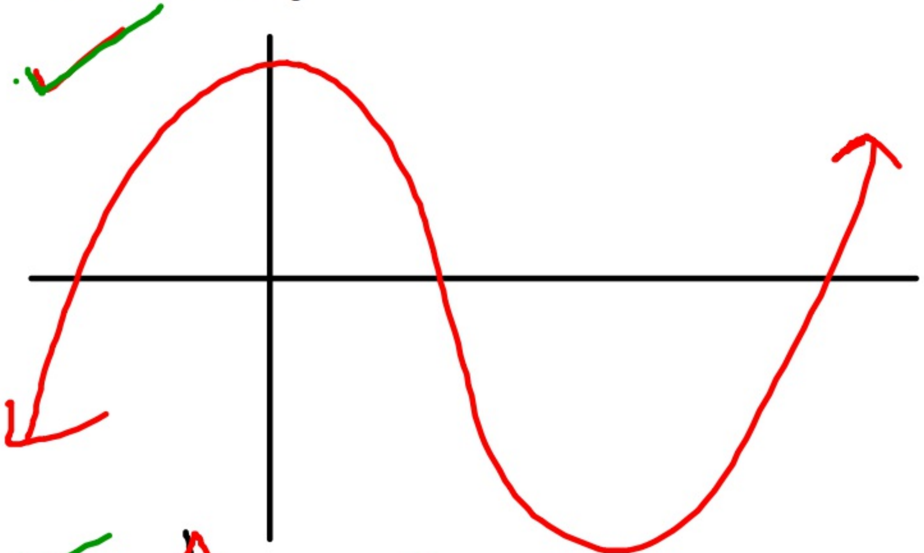
FACT: Calculus is about things that change and how change occurs, on small and big scales.

$f(3)$ $\qquad \lim_{x \to 3} f(x)$ $\qquad \frac{d}{dx}$ $\qquad \int$

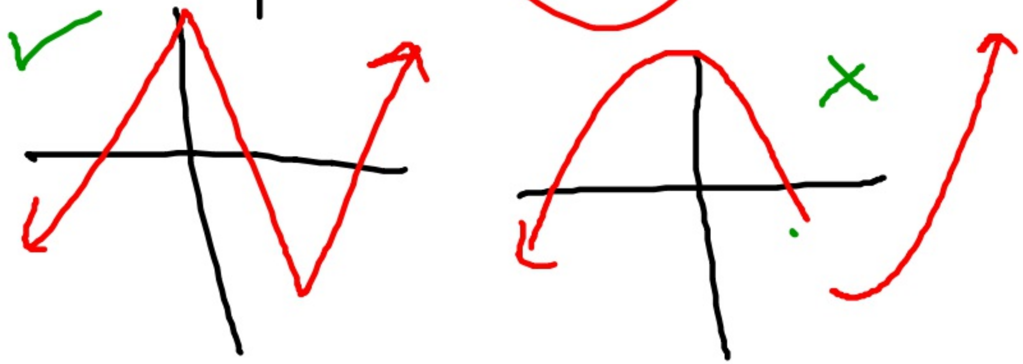
Our limits toolkit

- direct substitution
- factor/cancel
- rationalize w/ conjugate
- take one-sided limit(s)
- use degrees/dominance
- use special trig rules
- rewrite as piecewise

Continuity



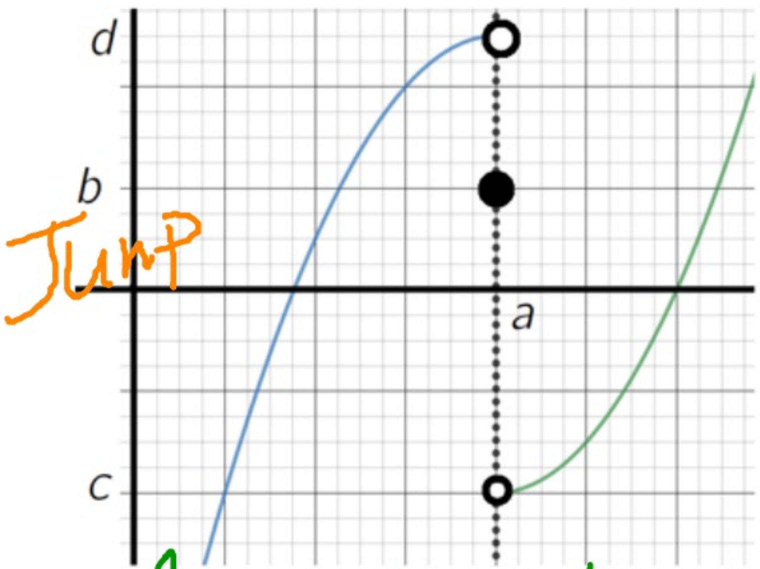
"a function you can draw in one stroke" (no pen pickup)



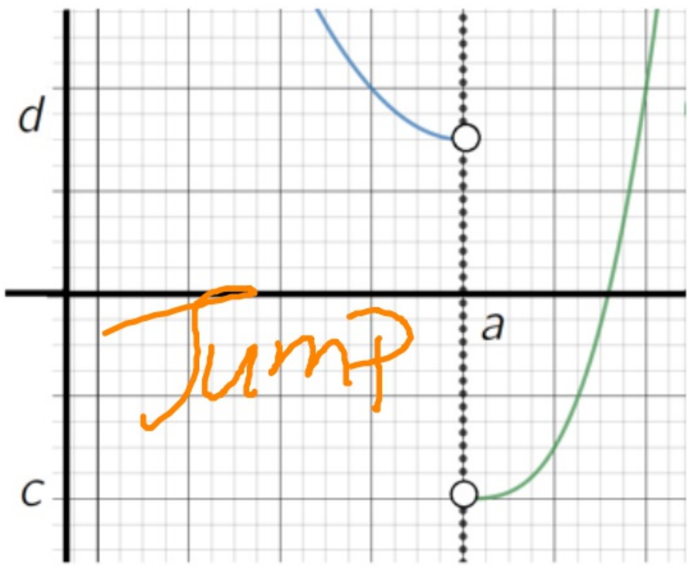
Continuity with roads and bridges

Work with your elbow partner
Write your answers in your notebook (in an organized way)

1. Yale University: $Y(x)$



2. Middlebury College: $M(x)$



$$\lim_{x\to a^-} M(x) = d$$

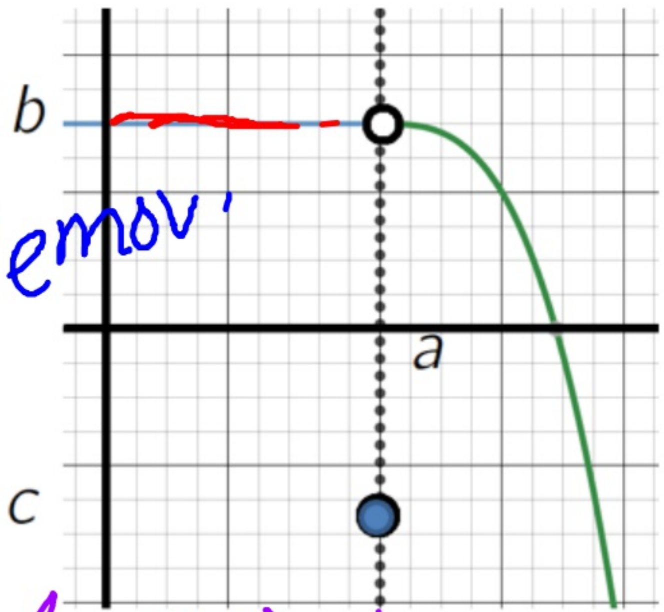
$$\lim_{x\to a^+} M(x) =$$

$$\lim_{x\to a^-} Y(x) = d \qquad Y(a) = b$$

$$\lim_{x\to a^+} Y(x) = c$$

$$M(a) = \varnothing$$

3. University of Georgia: $G(x)$



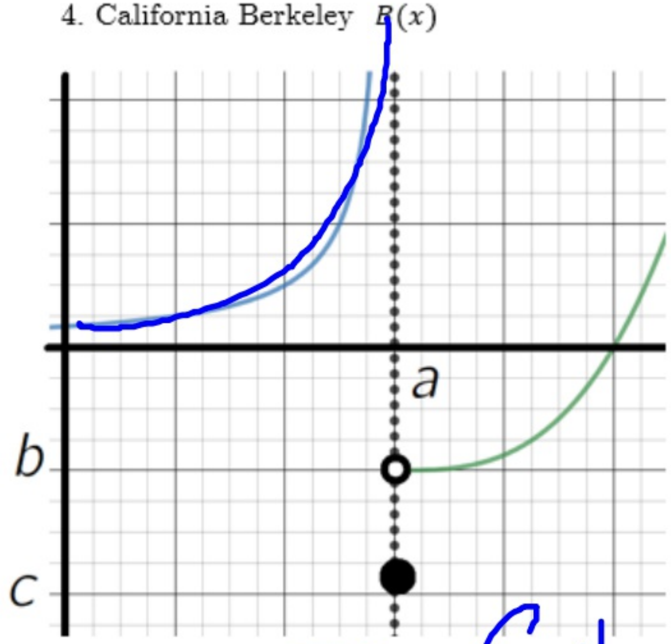
Remov'

$\lim_{x \to a^-} G(x) = b$

$\lim_{x \to a^+} G(x) = b$

$G(a) = c$

4. California Berkeley $B(x)$



$\lim_{x \to a^-} B(x) = \infty$

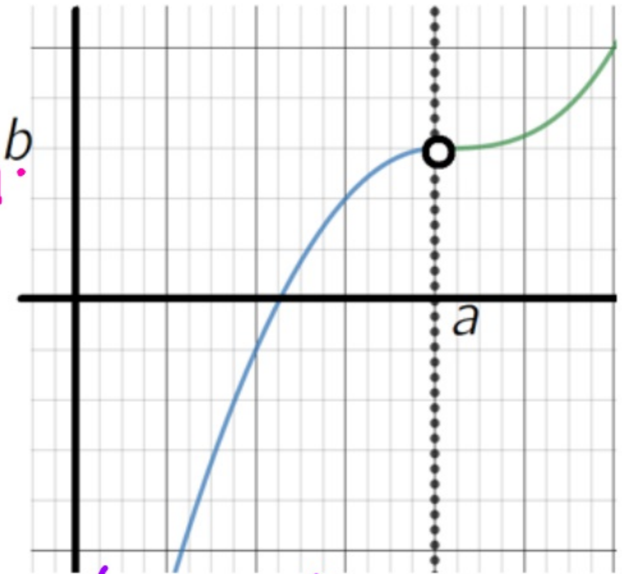
$\lim_{x \to a^+} B(x) = b$

$B(a) = c$

Infinite

5. University of North Carolina: $N(x)$

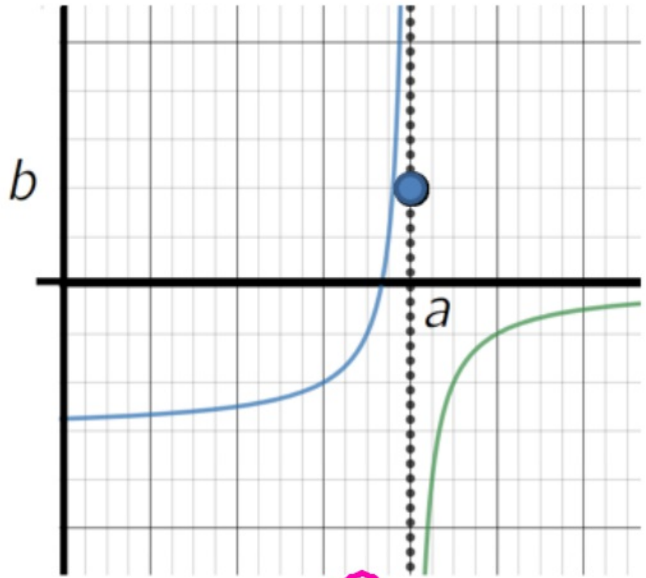
Rem.



$\lim_{x \to a} N(x) = b$

$N(a) = \emptyset$

6. University of Tennessee: $T(x)$



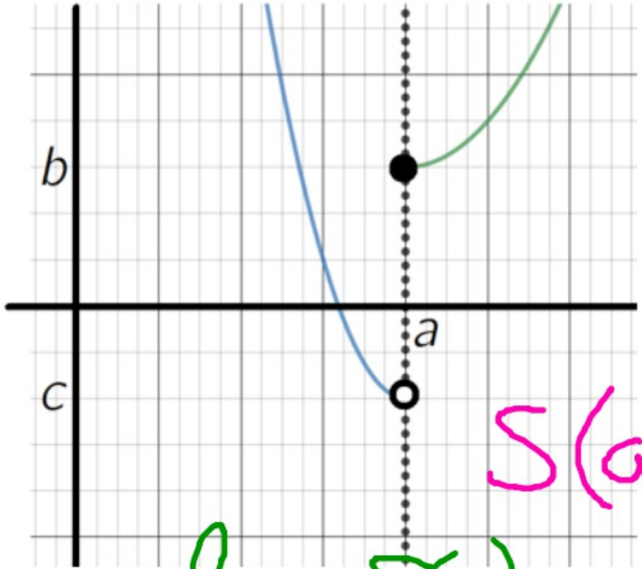
Inf.

$\lim_{x \to a^-} T(x) = \infty$

$\lim_{x \to a^+} T(x) = -\infty$

$T(a) = b$

7. Stanford University: $S(x)$

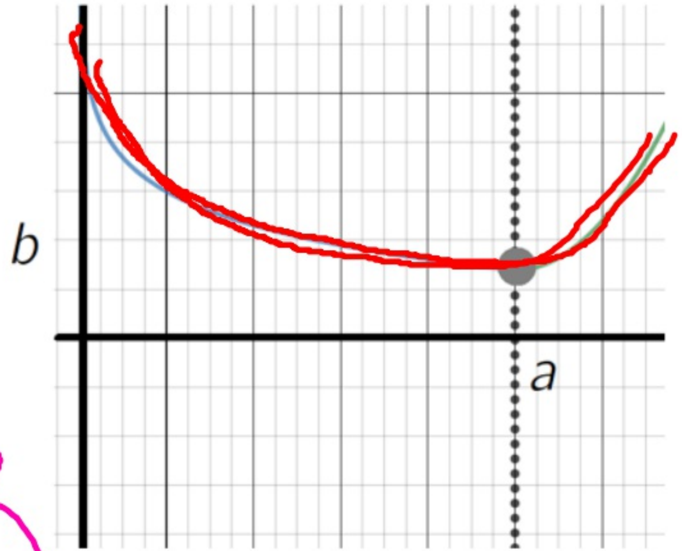


$S(a) = b$

$\lim_{x\to a^-} S(x) = c$

Jump. $\lim_{x\to a^+} S(x) = b$

8. Duke University: $D(x)$



$\lim_{x\to a^-} D(x) = b$

$\lim_{x\to a^+} D(x) = b$

$D(a) = b$

Continuous

Definition of Continuity at a Point

$\rightleftharpoons$ iff

A function *f(x)* is continuous at a point x=a if and only if

$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

"left road" "bridge" "right road"

<u>Alt:</u> $\lim_{x \to a} f(x) = f(a)$

Continuity on an Interval

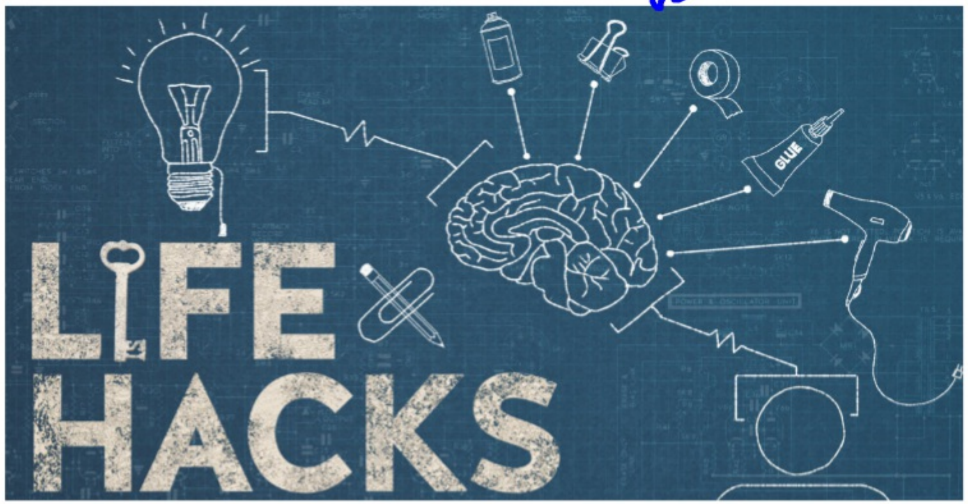
A function is continuous on an interval if it is continuous at every point *in* the interval

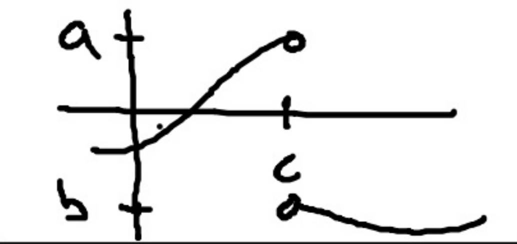
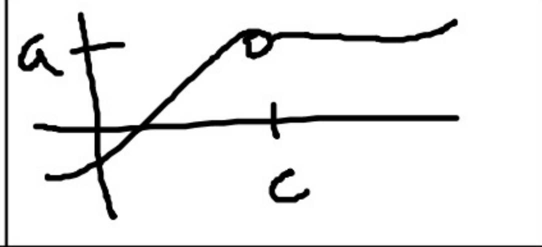
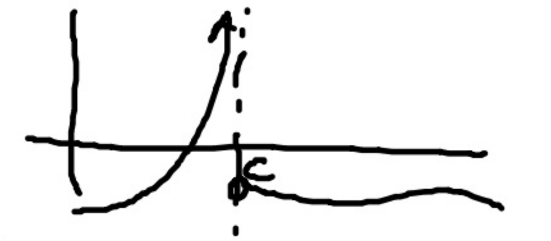
A function is continuous everywhere except where it is not

Save time:

- polynomial functions are always continuous everywhere
- piecewise functions often yield "jump" discontinuities
- sin(x) and cos(x) are always continuous everywhere
- rational functions often have v.a. or holes ("removable") discontinuities

(Inf.)



	Jump	Removable	Infinite
Graphically			
Algebraically	$\lim_{x\to c^-} f(x)=a$, $\lim_{x\to c^+} f(x)=b$, $a\neq b$ $(\in \mathbb{R})$	$\lim_{x\to c^-} f(x)=a$ $\lim_{x\to c^+} f(x)=a$ $f(c)\neq a$	$\lim_{x\to c^\pm} f(x)=\pm\infty$
Notes	piecewise functions; limit dne	"hole" limit exists!	identical to "vertical asymptote"

Discuss the continuity of f(x). Classify any discontinuities and justify with limits.

$$f(x)=\frac{x+1}{2x^2-6x-8}$$

$$\frac{x+1}{2(x^2-3x-4)}$$

$$\frac{x+1}{2(x+1)(x-4)}$$

$$\frac{1}{2(x-4)}$$

$x=-1$ is remov.

b/c $\lim_{x\to -1} f(x)=\frac{1}{2(-1-4)}=-\frac{1}{10}$

$x=4$ inf. disc.

b/c $\lim_{x\to 4^+} f(x)=\frac{1}{2(4^+-4)}=\frac{1}{2(0^+)}=\frac{1}{0^+}=\infty$

HW: p. 80 #39-57 (multiples of 3)

For each:

a. sketch a graph (use calc or desmos.com)

b. find and classify discontinuities algebraically (graph can help you find places to test with limits)