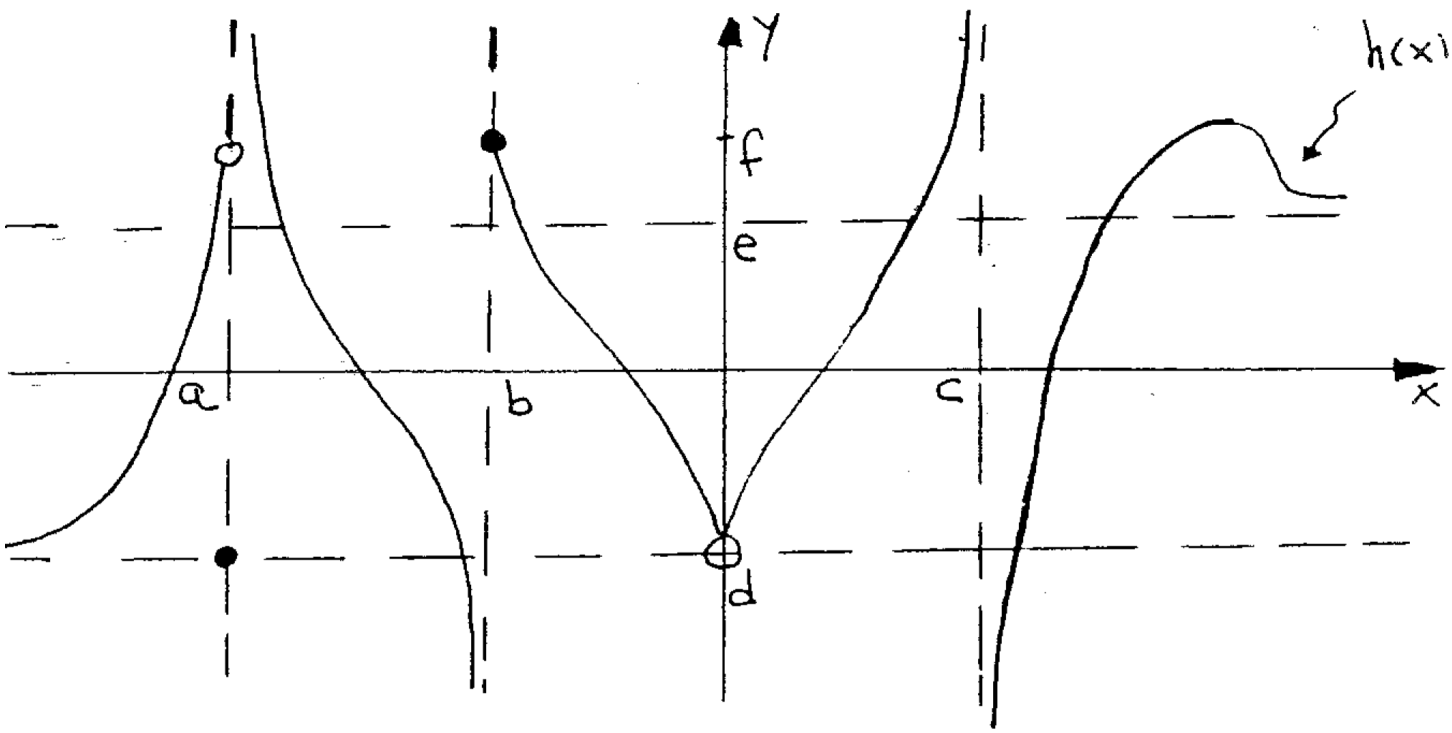
Practice Midterm

AP Calculus AB

Find each of the following limits. If the limit does not exist, explain why.

1. $\lim\limits_{x\to\infty} h(x) =$ e
2. $\lim\limits_{x\to-\infty} h(x) =$ d
3. $\lim\limits_{x\to a^+} h(x) =$ ∞
4. $\lim\limits_{x\to a^-} h(x) =$ f
5. $\lim\limits_{x\to a} h(x) =$ d.n.e. $\infty \neq f$
6. $\lim\limits_{x\to b} h(x) =$ dne: $-\infty \neq f$
7. $\lim\limits_{x\to 0} h(x) =$ d
8. $\lim\limits_{x\to b^+} h(x) =$ f



Evaluate each limit. If the limit does not exist, explain why using proper mathematical notation.

12. $\lim\limits_{x\to 5} \dfrac{x^2+3x-40}{2x-10} = \dfrac{\cancel{(x-5)}(x+8)}{2\cancel{(x-5)}} \Rightarrow \lim\limits_{x\to 5} \dfrac{x+8}{2} = \dfrac{13}{2}$

13. $\lim\limits_{x\to 5^+} \dfrac{-4x+1}{2x-10} = \lim\limits_{x\to 5^+} \dfrac{-4x+1}{2(x-5)} = \dfrac{-4\cdot 5^+ +1}{2(5^+-5)} = \dfrac{-20^-+1}{2\cdot 0^+} = \dfrac{-19^- \leftarrow \text{neg}}{0^+ \leftarrow \text{pos}} \Rightarrow -\infty$

14. $\lim\limits_{x\to 5^-} \dfrac{2x-7}{2x-10} = \dfrac{2\cdot 5^- -7}{2(5^- -5)} = \dfrac{10^- -7}{2\cdot 0^-} = \dfrac{3^- \leftarrow \text{pos}}{0^- \leftarrow \text{neg}}\ -\infty$

15. $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \frac{x+1-1}{(x)(\sqrt{x+1}+1)} = \frac{\cancel{x}}{\cancel{(x)}(\sqrt{x+1}+1)} \Rightarrow \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{0+1}+1} = \frac{1}{1+1} = \frac{1}{2}$

16. $\lim_{x\to\infty} \frac{-5+3x}{4-3x} = \frac{3}{-3} = -1$

H.A.

17. $\lim_{x\to -\infty} \frac{-5x^2-2x-3}{-23x-11+6x^4} = 0$

H.A.

18. $\lim_{x\to\infty} \frac{5x^6-2x-1}{3-7x^3-x+2x^5} = \infty$

H.A.?

19. Find the equation(s) of the vertical asymptotes of f(x)=$\frac{2x-8}{4x^2-64}$. Justify your answer. (with limits)

$\frac{2(x-4)}{4(x^2-16)} = \frac{2\cancel{(x-4)}}{4\cancel{(x-4)}(x+4)} = \frac{2}{4(x+4)}$

Removable, not inf.

V.A. @ $x=-4$

$\lim_{x\to -4^-} \frac{2}{4(x+4)} = \frac{2}{4(-4^-+4)} = \frac{2}{4(0^-)} \quad \frac{2}{0^-} = -\infty$

20. Find the equation(s) of the $\boxed{\text{horizontal asymptotes}}$ of f(x) below. Justify with limits.

$\lim_{x\to\infty} f(x)$ and $\lim_{x\to -\infty} f(x)$

$$f(x) = \begin{cases} \frac{2x-3}{50-10x}, & \text{if } x > 5 \\ x^2-x-10, & \text{if } x = 5 \\ \frac{-3-2x^2-10x}{4x^2-11x-7}, & \text{if } x < 5 \end{cases}$$

$\lim_{x\to\infty} \frac{2x-3}{50-10x} = \frac{2}{-10} = -1/5$

$\lim_{x\to -\infty} \frac{-3-2x^2-10x}{4x^2-11x-7} = \frac{-2}{4} = -1/2$

HA: $y = -\frac{1}{5}$ and $y = -\frac{1}{2}$

21. Find the values of a and b so that g(x) is everywhere continuous.

$$g(x) = \begin{cases} x^2, & if\ x < 2 \\ ax+b, & if\ 2 \le x \le 5 \\ 3x+1, & if\ x > 5 \end{cases}$$

Cont. @ $x=2$

$\lim_{x\to 2^-} g(x) = g(2) = \lim_{x\to 2^+} g(x)$

$(2)^2 = 2a+b = 2a+b$

$\Downarrow$

$4 = 2a+b$

Cont @ $x=5$

$\lim_{x\to 5^-} g(x) = g(5) = \lim_{x\to 5^+} g(x)$

$5a+b = 5a+b = 3(5)+1$

$\Downarrow$

$5a+b=16$

Elimination

$2a+b=4$

$-(5a+b=16)$

$-3a = -12$

$a=4$

$2(4)+b=4$

$8+b=4$

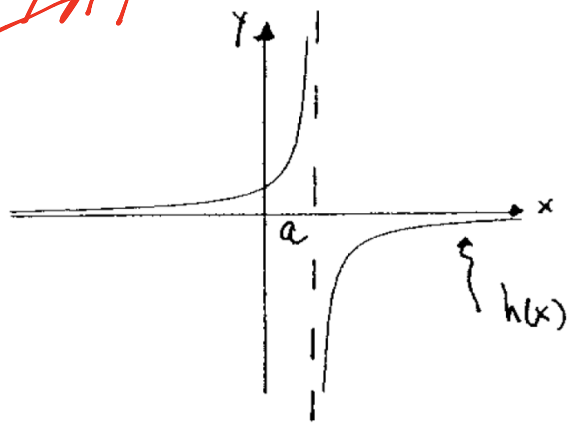
$b=-4$

$a=4$

$b=-4$

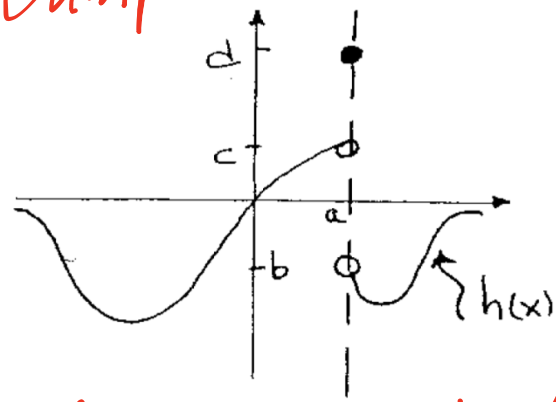
For 22-25, classify as jump, removable, or infinite discontinuities. Use limits to justify your answers.

22. Inf.



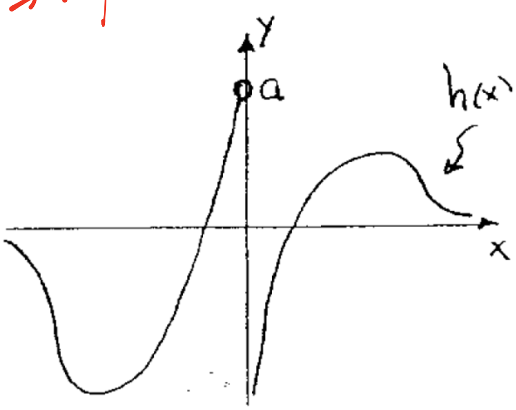
$\lim_{x\to a^+} h(x) = -\infty$

23. Jump



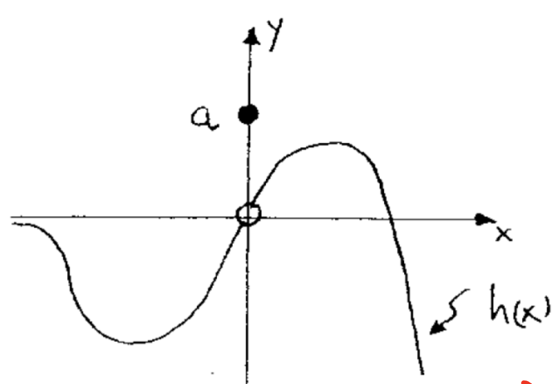
$\lim_{x\to a^-} h(x) = c \neq d = \lim_{x\to a^+} h(x)$

24. Inf.



$\lim_{x\to a^+} h(x) = -\infty$

25. Rem.



$\lim_{x\to 0} h(x) = 0 \neq h(0) = a$

26. Explain why $f(x) = -2x^2 - 5x + 6$ has a root in the interval [0,1].

$f(x)$ is cont. $\mathbb{R}$ b/c polynomial.

$f(0) = 6$

$f(1) = -2(1)^2 - 5 \cdot 1 + 6 = -1$

By IVT, there exists some $c \in (0,1)$ such that $f(c) = 0$ because

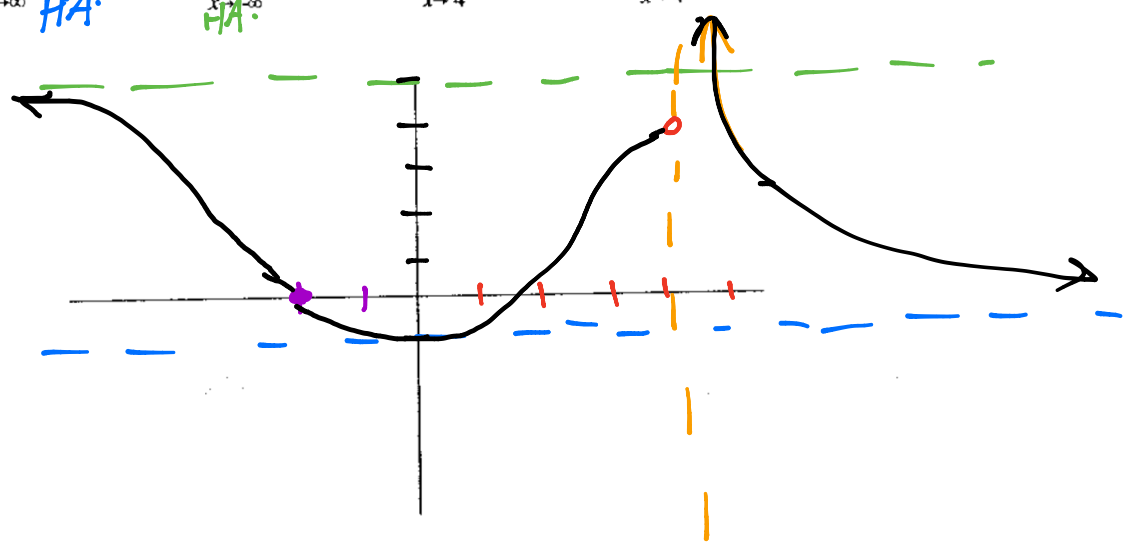
$6 \geq 0 \geq -1$

$f(0) \geq f(c) \geq f(1)$.

27. Graph a possible graph for the function H(x) satisfying the following conditions:

VA.

$\lim_{x\to\infty} H(x) = -1$ HA. $\quad \lim_{x\to-\infty} H(x) = 5$ HA. $\quad \lim_{x\to 4^+} H(x) = 4 \quad \lim_{x\to 4^-} H(x) = \infty$ and $H(-2) = 0$



Bonus: Find

$$\lim_{x\to\infty} \frac{\frac{-5+3x}{4-3x}}{\frac{5x^6-2x-1}{3-7x^3-x+2x^5}}$$

$$\frac{\lim_{x\to\infty} \frac{-5+3x}{4-3x}}{\lim_{x\to\infty} \frac{5x^6-2x-1}{3-7x^3-x+2x^5}} = \frac{-1}{\infty} = 0$$