$\lim_{x \to 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

$\lim_{n \to \infty} \frac{4n^2}{n^2 + 10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.

(B) $f(x)$ is continuous at $x = a$.

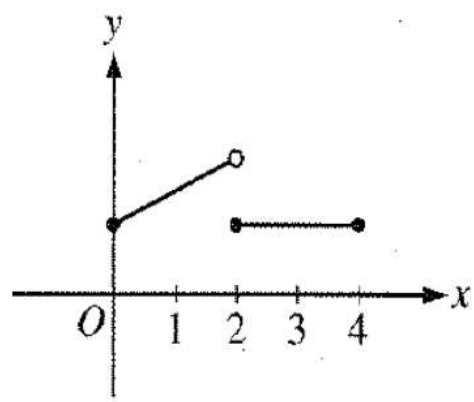
(C) $f(x)$ is defined at $x = a$.

(D) $f(a) = L$

(E) None of the above

$\lim_{x \to \infty} \left(1 + 5e^x\right)^{\frac{1}{x}}$ is

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent



Graph of f

The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

If k is a positive integer, then $\lim\limits_{x\to+\infty} \dfrac{x^k}{e^x}$ is

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

If $\lim\limits_{x\to 3} f(x) = 7$, which of the following must be true?

I. f is continuous at $x = 3$.
II. f is differentiable at $x = 3$.
III. $f(3) = 7$

(A) None (B) II only (C) III only

(D) I and III only (E) I, II, and III

The graph of which of the following equations has $y = 1$ as an asymptote?

(A) $y = \ln x$ (B) $y = \sin x$ (C) $y = \dfrac{x}{x+1}$ (D) $y = \dfrac{x^2}{x-1}$ (E) $y = e^{-x}$

Let f be the function defined by the following.

$$f(x) = \begin{cases} \sin x, & x < 0 \\ x^2, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ x - 3, & x \ge 2 \end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

$\lim\limits_{n\to\infty} \dfrac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) –5 (B) –2 (C) 1 (D) 3 (E) nonexistent

$\lim_{\theta \to 0} \dfrac{1-\cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

Which of the following functions are continuous at $x = 1$?

I. $\ln x$

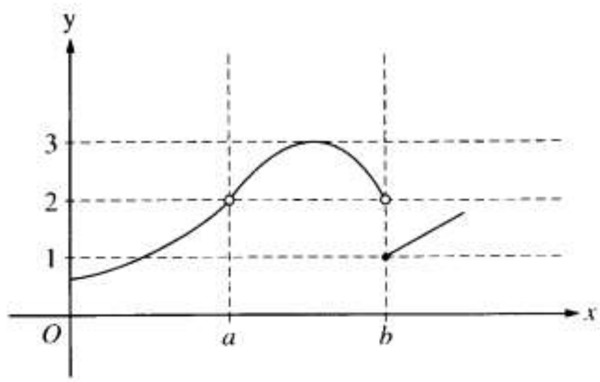
II. e^x

III. $\ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

If $f(x) = 2x^2 + 1$, then $\lim_{x \to 0} \dfrac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

$\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim\limits_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

If $a \ne 0$, then $\lim\limits_{x \to a} \dfrac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \ne 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{3}$ (D) 1 (E) $\dfrac{7}{5}$

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \ge g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h) = \dfrac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h) = \dfrac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim\limits_{x \to h} g(x) = g(h)$.