Definition of continuity at a point

Removable Disc.

$$\lim_{x \to a} f(x) = T \text{, where } T \in \mathbb{R}$$

but $f(a) \neq T$.

Continuity @ a point

A function $f(x)$ is continuous at $x = c$ if all are true:

"Roads meet"

show left = right (I) $\lim_{x \to c} f(x) = a,\ a \in \mathbb{R}$.

plug c in (II) $f(c)$ is defined "there's a bridge"

show equality (III) $\lim_{x \to c} f(x) = f(c) = a$ "roads meet bridge"

Finding a value to make a function continuous

63. $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

$\lim_{x \to 2^-} x^3 = 8 \qquad f(2) = 8$

$\lim_{x \to 2^+} ax^2 = 4a \qquad 8 = 4a$

$a = 2$

62. $f(x) = \begin{cases} 3x^3, & x \leq 1 \\ ax+5 & x > 1 \end{cases}$

(1)

$\lim_{x \to 1^-} 3x^3 = 3$

$f(1) = 3(1)^3 = 3$

$\lim_{x \to 1^+} ax+5 = a+5$

$3 = a+5$

$-5 \quad -5$

$-2 = a$

Find a value to make a function continuous

2) $g(x) = \begin{cases} \frac{16-x^2}{8-2x}, & x \neq 4 \\ k & x = 4 \end{cases}$

$$\frac{(4+x)\cancel{(4-x)}}{2\cancel{(4-x)}} = \frac{4+x}{2} \Rightarrow \frac{4+4}{2} \quad \frac{8}{2} = 4$$

$k = 4$

Showing/justifying continuity

(1) $\lim_{x \to 5} f(x)$ → $\lim_{x \to 5^-} x^2 - 5x - 4 = -4$
→ $\lim_{x \to 5^+} -4 = -4$

$\lim_{x \to 5} f(x) = -4$

(2) $f(5) = -4$

(3) $\underbrace{\lim_{x \to 5} f(x) = -4}_{(1)} = \underbrace{f(5)}_{2}$ Yes!

Classifying discontinuities

$$p(x) = \frac{3(x+5)}{2(x^2+3x-10)} = \frac{3\cancel{(x+5)}}{2\cancel{(x+5)}(x-2)} = \frac{3}{2(x-2)}$$

1) $x=-5$, $x=2$

2) $\underline{x=-5}$ is rem.

$\lim_{x\to -5} p(x) = -\frac{3}{14}$ $\quad p(-5)$ undef.

$\underline{x=2}$ Inf.

$$\lim_{x\to 2^+} \frac{3}{2(x-2)}$$

$$\frac{3}{2(2^+-2)}$$

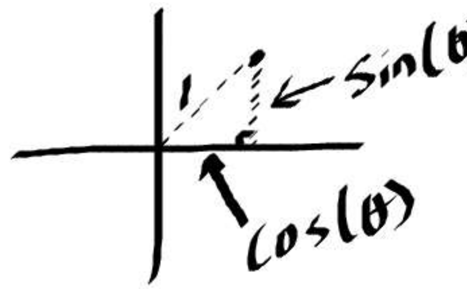
$$\frac{3}{2\,0^+}$$

$$\frac{3}{0^+} = \infty$$

Miscellaneous AP problems

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$\sin^2 = 1-\cos^2$



$$\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$$

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{2(1-\cos^2\theta)} \to \lim_{\theta\to 0}\frac{\cancel{1-\cos\theta}}{2\cancel{(1-\cos\theta)}(1+\cos\theta)}$$

(a^2-b^2)

$(a+b)(a-b)$

$$\lim_{\theta\to 0}\frac{1}{2(1+\cos\theta)}$$

$$\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2}$$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$\sin^2 x = (\sin x)^2$

$\sin^2 x \neq \sin x^2$

$$\lim_{x\to 0} \frac{\sin^2(2x)}{x^2}$$

$$\lim_{x\to c} (f(x))^n = \left(\lim_{x\to c} f(x)\right)^n$$

$$\frac{a^2}{b^2} = \left(\frac{a}{b}\right)^2$$

$$\lim_{x\to 0} \left(\frac{\sin(2x)}{x}\right)^2$$

$$\left(\lim_{x\to 0} \frac{2\sin(2x)}{2x}\right)^2$$

$$\lim_{x\to c} 2f(x) = 2\lim_{x\to c} f(x)$$

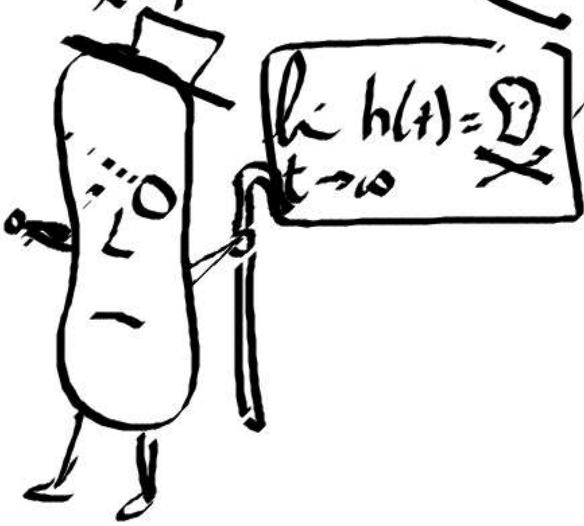
$$\lim_{x\to 3} 2x^2 = 2\lim_{x\to 3} x^2$$

$$\left(2\lim_{2x\to 0} \frac{\sin 2x}{2x}\right)^2$$

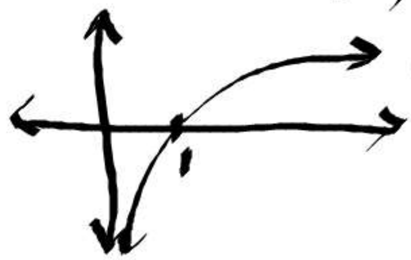
as $x\to 0$,
$2x \to 0$

$$\lim_{x\to 1} \frac{x}{\ln x}$$

$$\lim_{x\to 1^-} \frac{x}{\ln x} = \frac{1^-}{\ln(1^-)} = \frac{1^-}{0^-} = -\infty$$



$$\lim_{x\to 1^+} \frac{x}{\ln x} = \frac{1^+}{\ln(1^+)} = \frac{1^+}{0^+} = \infty$$



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