D-AD2 Practice Assessment SOLUTIONS

For each function below, find the derivative function.

1. $f(x) = 4\sqrt[3]{x^2} + 2x - \frac{1}{x}$

   Rewrite: $f(x) = 4x^{\frac{2}{3}} + 2x - x^{-1}$

   Power rule: $f'(x) = 4 * \frac{2}{3}x^{-\frac{1}{3}} + 2 - -1x^{-2}$ → *Simplify*: $f'(x) = \frac{8}{3}x^{-\frac{1}{3}} + 2 + \frac{1}{x^2}$ → $\boxed{f'(x) = \frac{8}{3x^{1/3}} + 2 + \frac{1}{x^2}}$

2. $g(t) = -2\cos t$

   $g'(t) = -2 * -\sin t$ → $\boxed{g'(t) = 2\sin t}$

3. $y = 5^x + \csc x - \tan x$

   $\boxed{y' = 5^x * \ln 5 - \csc x \cot x - \sec^2 x}$ *Just do it rule by rule*

4. $s(t) = e^{3t}$

   $s'(t) = e^{3t} * 3$ → $\boxed{s'(t) = 3e^{3t}}$

   (exponential derivative + chain rule)

D-AD2b

5. Find $\frac{dy}{dx}$ if $y = \sec^{-1} x$ Not on 10/26 test

   $\boxed{\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}}$ *Directly from formula booklet*

6. If $y = \ln(5x+1)$, find $\frac{dy}{dx}\big|_{x=1}$

   Note that this problem has you find the numerical slope value, so plug x=1 in after finding dy/dx.

   $\frac{dy}{dx} = \frac{1}{5x+1} * 5$ → $\frac{dy}{dx} = \frac{5}{5x+1}$ Now plug x=1 in: $\boxed{\frac{dy}{dx} = \frac{5}{5(1)+1} = \frac{5}{6}}$

   (natural log rule and chain rule)

7. Find the derivative of $y = \tan(3x^2 - 3)$

   $y' = \sec^2(3x^2-3) * 6x$ → $\boxed{y' = 6x\sec^2(3x^2-3)}$

   (trig derivative and chain rule)

D-AD3

Let f(x)=$5x^2 - 3x + 5$ and g(x)=$x^2 + \cos x$

8. If $h(x) = f(x)g(x)$, find $h'(x)$. [No need to simplify.]

   $h(x) = (5x^2 - 3x + 5)(x^2 + \cos x)$

   List out ingredients: $f: 5x^2 - 3x + 5$ $g: x^2 + \cos x$

   $f' = 10x - 3$ $g': 2x - \sin x$

   Product Rule recipe: $f'g + fg'$

   $\boxed{h'(x) = (10x-3)(x^2+\cos x) + (5x^2-3x+5)(2x - \sin x)}$

9. If $p(x) = \frac{f(x)}{g(x)}$, find $p'(x)$ [No need to simplify]

   List out ingredients: $f: 5x^2 - 3x + 5$ $g: x^2 + \cos x$

   $f' = 10x - 3$ $g': 2x - \sin x$

   Quotient Rule recipe: $\frac{f'g - fg'}{g^2}$ $\boxed{\frac{(10x-3)(x^2+\cos x)-(5x^2-3x+5)(2x-\sin x)}{(x^2+\cos x)^2}}$

D-AD4

10. Calculate the derivative of y=$\sqrt[3]{6x^2-3x+1}$

Rewrite: $y = (6x^2-3x+1)^{1/3}$

Chain rule: $y' = \frac{1}{3}(6x^2-3x+1)^{-\frac{2}{3}} * (12x-3)$

Simplify: $y' = \frac{12x-3}{3(6x^2-3x+1)^{\frac{2}{3}}} \rightarrow \boxed{y' = \frac{4x-1}{(6x^2-3x+1)^{\frac{2}{3}}}}$

11. If y=$\cos^2(3x-12)$, find $\frac{dy}{dx}$.

Rewrite: $y = [\cos(3x-12)]^2$

Chain rule....twice!! : $\frac{dy}{dx} = 2\,[\cos(3x-12)]^1 * -\sin(3x-12) * 3 \rightarrow \boxed{-6\sin(3x-12)\cos(3x-12)}$

12. Use the table to find $h'(1)$ if $h(x) = f(g(x))$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	3	-2
2	3	$\frac{3}{2}$	1	$-\frac{1}{2}$
3	4	$-\frac{1}{2}$	2	$\frac{3}{2}$
4	2	-2	4	2

Chain rule by definition:

$$h'(x) = f'(g(x)) * g'(x)$$
$$h'(1) = f'(g(1)) * g'(1)$$
$$h'(1) = f'(3) * -2$$
$$h'(1) = -\frac{1}{2} * -2 \rightarrow \boxed{1}$$

D-CD4

13. Show that $f(x) = \begin{cases} 5x^2-3x-6 & x \le 1 \\ -2x^2-2 & x > 1 \end{cases}$ is not differentiable at x=1.

Check continuity at x=1

From left and at 1: $f(1^-) = 5(1)^2 - 3(1) - 6 \;\rightarrow -4$

From right $f(1^+) = -2(1^2) - 2 \rightarrow -2-2 \rightarrow -4$ $-4 = -4$, so yes continuous

Check differentiability at x=1

$f'(x) = \begin{cases} 10x-3 & x \le 1 \\ -4x & x > 1 \end{cases}$

Slope from left: $f'(1^-) = 10(1) - 3 = 7$

Slope from right: $f(1^+) = -4(1) = -4$ $7 \neq -4$ so, not differentiable at x=1.

14. Find the values of a and b that would make f(x) differentiable. $f(x) = \begin{cases} ax^2+bx-2 & x \le 2 \\ -2x^2+2x+8 & x > 2 \end{cases}$

Continuous? Check 2 from left, right, and exactly 2

Left and middle = Right

$a(2^2) + b(2) - 2 = -2(2^2) + 2(2) + 8$

$4a + 2b - 2 = -8 + 4 + 8$

$\mathbf{4a + 2b = 6}$

Differentiable? Check 2 from left, right, and exactly 2 of $f'(x)$

$f'(x) = \begin{cases} 2ax+b & x \le 2 \\ -4x+2 & x > 2 \end{cases}$

Left and middle = Right

$2a(2) + b = -4 * 2 + 2$

$\mathbf{4a + b = -6}$

$- \begin{cases} \mathbf{4a + 2b = 6} \\ \mathbf{4a + b = -6} \end{cases}$ Solve system of equations

$b = 12$

$4a + 12 = -6$

$4a = -18$

$a = -\frac{18}{4}$

15. Use the axes below to sketch a continuous function with a corner at x=3, a cusp at x=0, and a vertical tangent at x=−2.

Possible answer:
See notes + handout on cusps, corners, and vertical tangents