Derivative BINGO

B	I	N	G	O
$2x-\frac{2}{x^3}$	$2x-\frac{2}{x^2}+\frac{15}{x^4}$	$4x^3+4x$	0	$\frac{1}{(x+1)^2}$
$2x+\frac{3}{4\sqrt[4]{x^7}}$	$5x^4-2x$	$2x(3x^2)$	$8x+12$	$8x^3+16x$
$3x^2-8x-3$	$2x+x^{-2}$	$\frac{-4x}{(x^2-1)^2}$	$12x-7$	$\frac{2-6x^2}{(3x^2+1)^2}$
$15(x^2-x^4)$	$3x^2-6x$	$2x(1-x^2)^{-2}$	$\frac{-19}{(3x-2)^2}$	$9x^2+x^3-x^4$
$12x+13$	$\frac{1-x^2}{(x^2+1)^2}$	$2x+1$	$(3x-5)\frac{1}{2\sqrt{x}}+3\sqrt{x}$	$\frac{1}{2\sqrt{x}}+2x$

Match the functions in the boxes (first derivatives) with the number of the functions below. Try to get a BINGO!

1. $y=(x^2+1)^2$
2. $y=(3x-1)(2x+5)$
3. $y=\sqrt{x}+x^2$
4. $y=x^3-3x^2+2$
5. $y=\frac{2x+5}{3x-2}$
6. $y=(2x^2+2)(x^2+3)$
7. $y=\frac{6x^3-7x^2}{x}$
8. $y=\frac{x^2+1}{x^2-1}$
9. $y=x^3-4x^2-3x$
10. $y=5x^3-3x^5$
11. $y=(2x+3)^2$
12. $y=(x+x^{-1})^2$
13. $y=\frac{2x}{3x^2+1}$
14. $y=3x\left(\frac{x^3}{2}\right)$
15. $y=x^2(x^3-1)$
16. $y=\frac{x}{x^2+1}$
17. $y=\frac{x^3-1}{x}$
18. $y=\frac{1}{1-x^2}$
19. $y=\frac{-1}{x+1}$
20. $y=(x-2)(x+3)$
21. $y=\pi^3+4\pi$
22. $y=x^2-\frac{1}{\sqrt[4]{x^3}}$
23. $y=(3x-5)\sqrt{x}$
24. $y=3x^3+\frac{1}{4}x^4-\frac{1}{5}x^5$
25. $y=\frac{x^5+2x^2-5}{x^3}$

Derivatives Review

In Exercises 1–30, find the derivative of the function.

1. $y=x^5-\frac{1}{8}x^2+\frac{1}{4}x$

2. $y=3-7x^3+3x^7$

3. $y=2\sin x\cos x$

4. $y=\frac{2x+1}{2x-1}$

5. $s=\cos(1-2t)$

6. $s=\cot\frac{2}{t}$

7. $y=\sqrt{x}+1+\frac{1}{\sqrt{x}}$

8. $y=x\sqrt{2x+1}$

9. $r=\sec(1+3\theta)$

10. $r=\tan^2(3-\theta^2)$

11. $y=x^2\csc 5x$

12. $y=\ln\sqrt{x}$

13. $y=\ln(1+e^x)$

14. $y=xe^{-x}$

15. $y=e^{(1+\ln x)}$

16. $y=\ln(\sin x)$

17. $r=\ln(\cos^{-1}x)$

18. $r=\log_2(\theta^2)$

19. $s=\log_5(t-7)$

20. $s=8^{-t}$

21. $y=x^{\ln x}$

22. $y=\frac{(2x)2^x}{\sqrt{x^2+1}}$

23. $y=e^{\tan^{-1}x}$

24. $y=\sin^{-1}\sqrt{1-u^2}$

25. $y=t\sec^{-1}t-\frac{1}{2}\ln t$

26. $y=(1+t^2)\cot^{-1}2t$

27. $y=z\cos^{-1}z-\sqrt{1-z^2}$

28. $y=2\sqrt{x-1}\csc^{-1}\sqrt{x}$

29. $y = \csc^{-1}(\sec x),\ 0 \le x \le 2\pi$

30. $r = \left(\dfrac{1 + \sin\theta}{1 - \cos\theta}\right)^2$

In Exercises 31–34, find all values of x for which the function is differentiable.

31. $y = \ln x^2$

32. $y = \sin x - x\cos x$

33. $y = \sqrt{\dfrac{1 - x}{1 + x^2}}$

34. $y = (2x - 7)^{-1}(x + 5)$

In Exercises 35–38, find dy/dx.

35. $xy + 2x + 3y = 1$

36. $5x^{4/5} + 10y^{6/5} = 15$

37. $\sqrt{xy} = 1$

38. $y^2 = \dfrac{x}{x + 1}$

In Exercises 39–42, find d^2y/dx^2 by implicit differentiation.

39. $x^3 + y^3 = 1$

40. $y^2 = 1 - \dfrac{2}{x}$

41. $y^3 + y = 2\cos x$

42. $x^{1/3} + y^{1/3} = 4$

In Exercises 43 and 44, find all derivatives of the function.

43. $y = \dfrac{x^4}{2} - \dfrac{3}{2}x^2 - x$

44. $y = \dfrac{x^5}{120}$

In Exercises 45–48, find an equation for the **(a)** tangent and **(b)** normal to the curve at the indicated point.

45. $y = \sqrt{x^2 - 2x},\quad x = 3$

46. $y = 4 + \cot x - 2\csc x,\quad x = \pi/2$

47. $x^2 + 2y^2 = 9,\quad (1, 2)$

48. $x + \sqrt{xy} = 6,\quad (4, 1)$

In Exercises 49–52, find an equation for the line tangent to the curve at the point defined by the given value of t.

49. $x = 2\sin t,\quad y = 2\cos t,\quad t = 3\pi/4$

50. $x = 3\cos t,\quad y = 4\sin t,\quad t = 3\pi/4$

51. $x = 3\sec t,\quad y = 5\tan t,\quad t = \pi/6$

52. $x = \cos t,\quad y = t + \sin t,\quad t = -\pi/4$

53. Writing to Learn

(a) Graph the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2. \end{cases}$$

(b) Is f continuous at $x = 1$? Explain.

(c) Is f differentiable at $x = 1$? Explain.

54. Writing to Learn For what values of the constant m is

$$f(x) = \begin{cases} \sin 2x, & x \le 0 \\ mx, & x > 0 \end{cases}$$

(a) continuous at $x = 0$? Explain.

(b) differentiable at $x = 0$? Explain.

In Exercises 55–58, determine where the function is **(a)** differentiable, **(b)** continuous but not differentiable, and **(c)** neither continuous nor differentiable.

55. $f(x) = x^{4/5}$

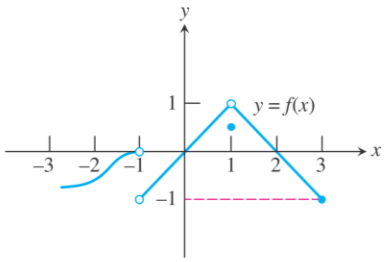
56. $g(x) = \sin(x^2 + 1)$

57. $f(x) = \begin{cases} 2x - 3, & -1 \le x < 0 \\ x - 3, & 0 \le x \le 4 \end{cases}$

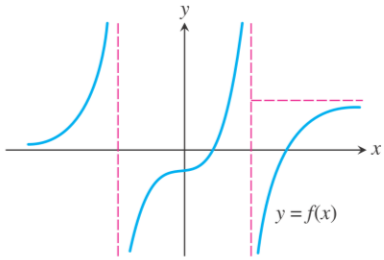
58. $g(x) = \begin{cases} \dfrac{x - 1}{x}, & -2 \le x < 0 \\ \dfrac{x + 1}{x}, & 0 \le x \le 2 \end{cases}$

In Exercises 59 and 60, use the graph of f to sketch the graph of f'.

59. *Sketching f′ from f*

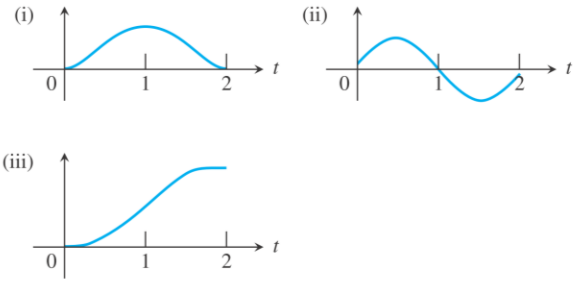


60. *Sketching f′ from f*



61. *Recognizing Graphs* The following graphs show the distance traveled, velocity, and acceleration for each second of a 2-minute automobile trip. Which graph shows

(a) distance **(b)** velocity? **(c)** acceleration?



62. *Sketching f from f′* Sketch the graph of a continuous function f with $f(0) = 5$ and

$$f'(x) = \begin{cases} -2, & x < 2 \\ -0.5, & x > 2. \end{cases}$$

63. *Sketching f from f′* Sketch the graph of a continuous function f with $f(-1) = 2$ and

$$f'(x) = \begin{cases} -2, & x < 1 \\ 1, & 1 < x < 4 \\ -1, & 4 < x < 6. \end{cases}$$