Derivatives of Inverses

- If f and g are inverses, and if f(a) = b, then what is g(b) = ? a

- Finding an inverse:

$$y = 2x^3 + 3$$

Find y^{-1}.

Switch x & y.

$$x = 2y^3 + 3$$

$$x - 3 = 2y^3$$

$$\frac{x-3}{2} = y^3$$

inverse of original

$$\boxed{\sqrt[3]{\frac{x-3}{2}} = y}$$

Derivative of Inverse

$f(x) = x\sqrt{x-3}$

"Find the derivative of the inverse of f at $a=4$."

Derivative of Inverse

if f and g are inverses, then

$$g'(x) = \frac{1}{f'(g(x))}$$

① Find x so that $f(x)=4$

$$\left(x\sqrt{x-3} = 4\right)^2$$

$$x^2(x-3) = 16$$

$$x^3 - 3x^2 - 16 = 0$$

solve w/ calc.

$$\underline{x=4}$$

$f(4)=4$

$g(4)=4$

② Find $f'(x)$.

$f = x\sqrt{x-3}$

$$f'(x) = \sqrt{x-3} + \frac{1}{2}x\cdot(x-3)^{-1/2}$$

③ Plug into formula.

$$g'(4) = \frac{1}{f'(g(4))}$$

$$g'(x) = \frac{1}{f'(4)} = \frac{1}{3}$$

Inverse Derivatives continued:

Let g be the inverse of f. Find g'(7) if $f(x) = 5-2x^3$

Set $f = 7$

$5-2x^3 = 7$

$\sqrt[3]{-1} = x$

$-1 = x$

$f:(-1,7)$

$g:(7,-1)$

$f'(x) = -6x^2$

$g'(x) = \dfrac{1}{f'(g(x))}$

$g'(7) = \dfrac{1}{f'(g(7))}$

$g'(7) = \dfrac{1}{f'(-1)} \Rightarrow \dfrac{1}{-6}$

Your Turn:

$[\sin(3x)]^2$

Let g be the inverse of f. Find g'(3) for f(x) $=\sqrt{x^2-3x-1}$ for x>4

$(x^2-3x-1)^{1/2}$

(alternate wording:

find the derivative of the inverse of the given f(x) at a=3.)

$f'(x) = \frac{1}{2}(x^2-3x-1)^{-1/2}\cdot(2x-3)$

$f'(x) = \frac{1\cdot(2x-3)}{2(x^2-3x-1)^{1/2}}$

$f'(5) = \frac{7}{2\cdot 3}$

$7/6$

$\sqrt{x^2-3x-1} = 3$

$x = 5, -2$

$f: (5,3)$

$g: (3,5)$

$g'(x) = \frac{1}{f'(g(x))}$

$g'(3) = \frac{1}{f'(\cancel{g(3)})} = \frac{1}{f'(5)} = \frac{1}{7/6} \sim 6/7$

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Homework:

p 344: 63-66, 88 (D-AD6)
Assesment on Friday

→ Implicit Diff (D-AD5)

→ exp/log deriv. (D-AD4)

Implicit Review

$f' = -3 \quad g' = \frac{dy}{dx}$

Find $\frac{dy}{dx}$ for $\frac{d}{dx}\left(2y^3 - \underbrace{3x}_{f}\underbrace{y}_{g} + x^2\right) = \frac{d}{dx}(4)$

$$6y^2\frac{dy}{dx} + \left(-3y + -3x \cdot \frac{dy}{dx}\right) + 2x = 0$$

$$6y^2\frac{dy}{dx} - 3y - 3x\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx}\left(6y^2 - 3x\right) = 3y - 2x$$

$$\frac{dy}{dx} = \frac{3y-2x}{6y^2-3x}$$

Problem Solving Time!

Find your role cards in the boxes

Runner: get chart paper and markers
Reporter: write problem down on scratch paper as well as present to class
Recorder: write it on the chart paper
Leader: assist with everything

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Group 1: #2
Group 2: #7
Group 3: #5
Group 5: #8
Group 6: #5