2.17 Dérivé, Derivado, Ableitung, Derivative

639. Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	-3	$\frac{1}{2}$
1	3	5	$\frac{1}{2}$	-4

Find the first derivatives of the following combinations at the given value of x.

a) $6f(x) - g(x)$ at $x = 1$

b) $f(x)g^2(x)$ at $x = 0$

c) $\dfrac{f(x)}{g(x)+1}$ at $x = 1$

d) $f(g(x))$ at $x = 0$

e) $g(f(x))$ at $x = 0$

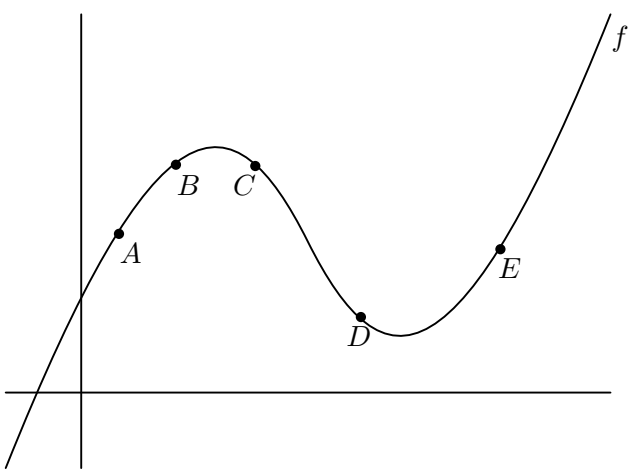
f) $(x + f(x))^{3/2}$ at $x = 1$

g) $f(x + g(x))$ at $x = 0$

640. If $x^2 - y^2 = 1$, find $\dfrac{d^2y}{dx^2}$ at the point $(2, \sqrt{3})$.

641. For what values of a and b will $f(x) = \begin{cases} ax & x < 2 \\ ax^2 - bx + 3 & x \geq 2 \end{cases}$ be differentiable for all values of x?

642. Use the graph of f to answer the following.



a) Between which two consecutive points is the average rate of change of the function greatest? Least?

b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

c) Sketch a tangent line to the graph between the points D and E such that the slope of the tangent is the same as the average rate of the change of the function between D and E.

d) Give a set of two points for which the average rate of change of the function is approximately equal to another set of two points.

643. The displacement from equilibrium of an object in harmonic motion on the end of a spring is $y = \frac{1}{3}\cos(12t) - \frac{1}{4}\sin(12t)$ where y is measured in feet and t is the time in seconds. Determine the position and velocity of the object when $t = \frac{\pi}{8}$.

644. The yield Y, in millions of cubic feet per acre, for a stand of timber at age t is $Y = 6.7e^{-48.1/t}$ where t is measured in years.

a) Find the limiting volume of wood per acre as t approaches infinity.

b) Find the rate at which the yield is changing when $t = 20$ years and $t = 60$ years.

645. Find expressions for the velocity and acceleration of a particle whose position is given by $x(t) = \sqrt{t} + \sin t$.

646. The position of a particle is given by $x(t) = t^3 - 9t^2 + 6t - 3$. Find the value of the position and velocity of the particle at the time when the acceleration is zero.

647. A ball thrown follows a path described by $y = x - 0.02x^2$.

a) Sketch a graph of the path.

b) Find the total horizontal distance the ball was thrown.

c) At what x-value does the ball reach its maximum height?

d) Find an equation that gives the instantaneous rate of change of the height of the ball with respect to the horizontal change. Evaluate this equation at $x = 0$, 10, 25, 30, and 50.

e) What is the instantaneous rate of change of the height when the ball reaches its maximum height?

648. A particle moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = \arctan t$.

a) Prove that the particle is always moving to the right.

b) Prove that the particle is always decelerating.

c) What is the limiting position of the particle as t approaches infinity?

649. The position at time $t \geq 0$ of a particle moving along a coordinate line is $x = 10\cos(t + \frac{\pi}{4})$.

a) What is the particle's starting position?

b) What are the points farthest to the left and right of the origin reached by the particle?

c) Find the particle's velocity and acceleration at the points in part (b).

d) When does the particle first reach the origin? What are its velocity, speed, and acceleration then?

No pain, no gain. —*Arnold Schwarzennegger*