Differentiability

A function f(x) is <u>differentiable</u> at a point if

(a) f(x) is continuous at that point

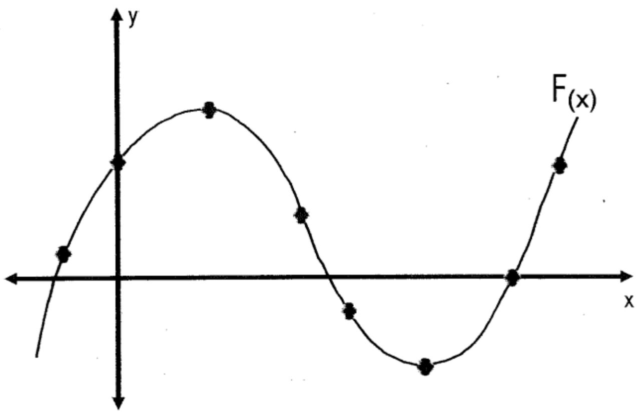
(b) f(x) has a unique tangent line with a defined slope

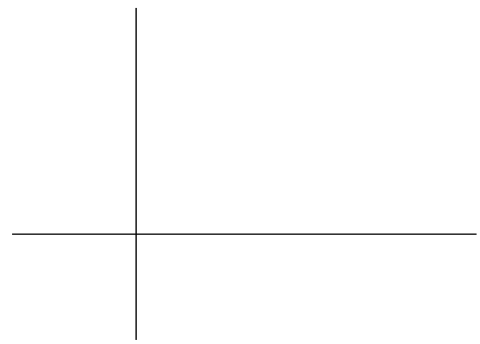
A function can be described as "differentiable" on an interval if it is differentiable at every point in that interval.

Another way to think of it: $f(x)$ is differentiable at c if and only if $f'(x)$ is continuous at c.

Task 1

With a ruler, draw tangent lines for the following function. Is it differentiable? ________



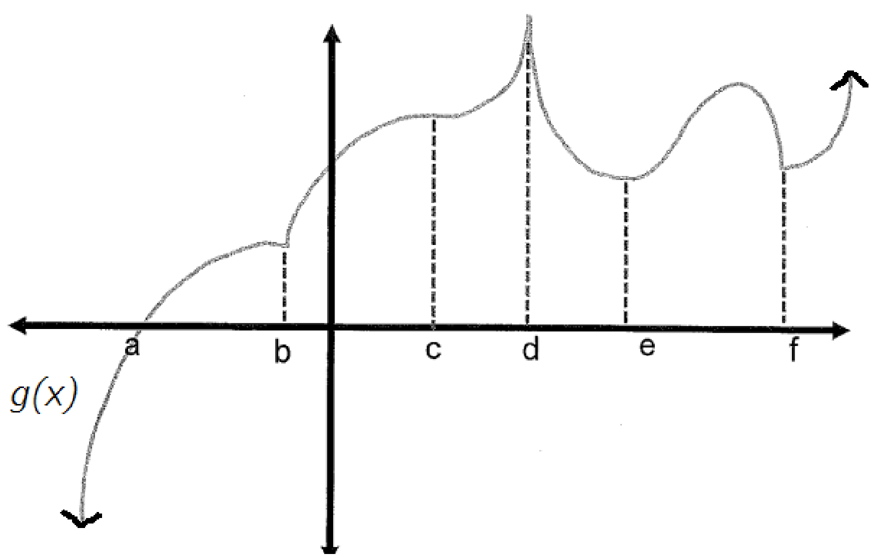


Sketch a discontinuous graph h(x) on the blank axes. Would you be able to draw tangent lines everywhere on h(x)? Explain.

This means that continuity (is/is not) a *necessary* condition for differentiability.

Task 2:

Below is a function g(x). Is it continuous?

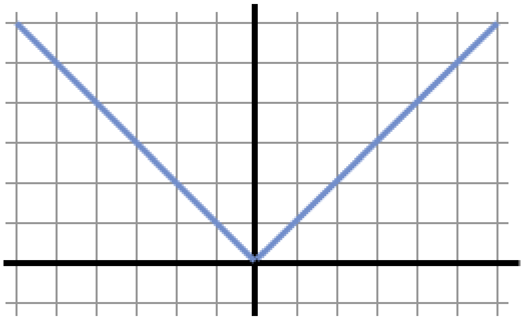


At what points is g(x) differentiable?

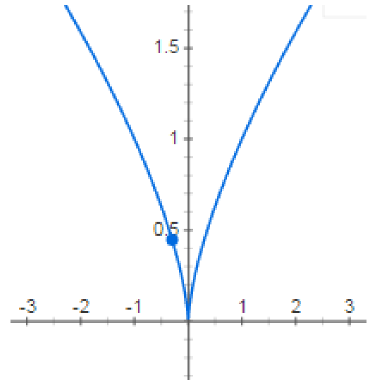
Where is it not differentiable?

This means continuity (is/is not) a *sufficient* condition for differentiability.

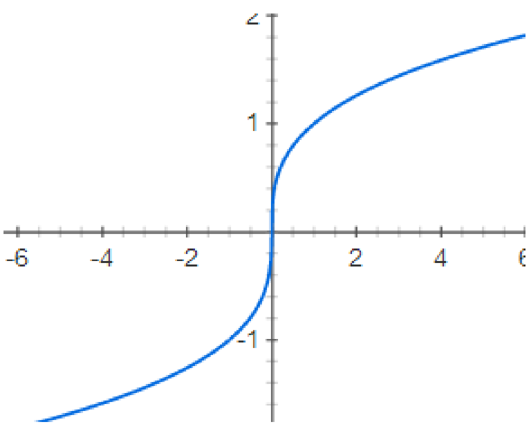
Below are 3 graphs of continuous functions. Algebraically find the derivative of each. Then examine the continuity of the derivative function at x=0.



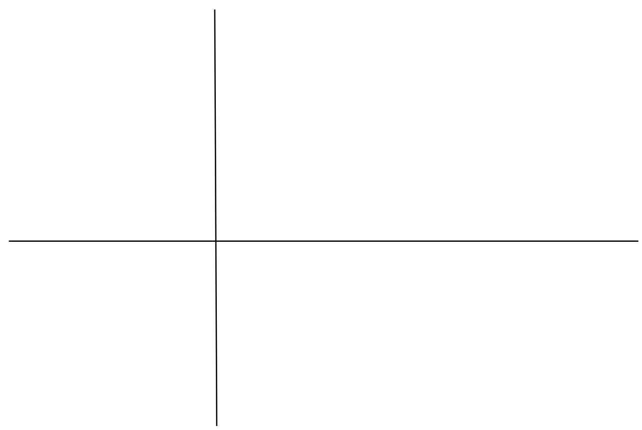
$f(x) = |x|$



$f(x) = x^{2/3}$



$f(x) = \sqrt[3]{x}$



Now sketch a generic function that is differentiable. What do you notice about its continuity and its derivative's continuity?

If f(x) is differentiable, then: