F-C4 NAME:

Consider $f(x) = -2x^2 + 18x - 28$

1. Is $f(x)$ continuous on the interval [5,9]? Explain.

yes. It's a polynomial and polynomials are continous everywhere.

2. Explain why there must be a value c in [5,9] such that $f(c) = 8$

$f(5) = -2(5)^2 + 18(5) - 28 = -50 + 90 - 28 = 40 - 28 = \underline{12}$

$f(9) = -2(9^2) + 18(9) - 28 = -162 + 162 - 28 = \underline{-28}$

$\begin{array}{r} 18 \\ \times 9 \\ \hline 162 \end{array}$

By the IVT, there must exist some c in $[5,9]$ such that $f(c) = 8$ because $f(5) \geq 8 \geq f(9)$.

3. Find the value or values of c in [5,9] such that $f(c) = 8$ guaranteed to exist in problem 2.

$f(c) = 8$

$-2c^2 + 18c - 28 = 8$

$0 = 2c^2 - 18c + 36$

$0 = 2(c^2 - 9c + 18)$

$0 = 2(c - 6)(c - 3)$

$c = 6$ ~~$c = 3$~~ not in [5,9]

D-C1

4. Use the limit definition of derivative to show that if $f(x) = 2x^2 - 3x + 5$, then $f'(x) = 4x - 3$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 3(x+\Delta x) + 5 - (2x^2 - 3x + 5)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x^2 + 2x\Delta x + \Delta x^2) - 3x - 3\Delta x + 5 - 2x^2 + 3x - 5}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + 2\Delta x^2 - \cancel{3x} - 3\Delta x + \cancel{5} - \cancel{2x^2} + \cancel{3x} - \cancel{5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 - 3\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(4x + 2\Delta x - 3)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 4x + 2\cancel{\Delta x} - 3$$

$$\boxed{f'(x) = 4x - 3}$$

D-C7

For each problem, find the derivative function.

5. $y = -3x^3 - 9x^5 + \frac{12}{5}x^4 + 2$

$\frac{dy}{dx} = -3 \cdot 3x^2 - 9 \cdot 5x^4 + \frac{12}{5} \cdot 4x^3 + 0$

$\boxed{-9x^2 - 45x^4 + \frac{48}{5}x^3}$

Power Rule

If $f(x) = x^n$
then $f'(x) = n \cdot x^{n-1}$

Negative Exponents

$\frac{a}{x^n} \rightleftarrows ax^{-n}$

Rational Exponents

$\sqrt[a]{x^b} \rightleftarrows x^{b/a}$

6. $f(x) = 4x^2 - \frac{4}{\sqrt[3]{x^2}} + 5\sqrt[5]{x^2}$

$f(x) = 4x^2 - 4x^{-2/3} + 5x^{2/5}$

$f'(x) = 4 \cdot 2x^1 - 4 \cdot -\frac{2}{3}x^{-5/3} + 5 \cdot \frac{2}{5}x^{-3/5}$

$\boxed{f'(x) = 8x + \frac{8}{3}x^{-5/3} + 2x^{-3/5}}$

or

$8x + \frac{8}{3\sqrt[3]{x^5}} + \frac{2}{\sqrt[5]{x^3}}$

7. $y = \frac{2}{x} + \sqrt{x} - \frac{1}{2x^4}$

$y = 2x^{-1} + x^{1/2} - \frac{1}{2} \cdot x^{-4}$

$\frac{dy}{dx} = 2 \cdot -1x^{-2} + \frac{1}{2}x^{-1/2} - \frac{1}{2} \cdot -4x^{-5}$

$\boxed{\frac{dy}{dx} = -2x^{-2} + \frac{1}{2}x^{-1/2} + 2x^{-5}}$

or

$-\frac{2}{x^2} - \frac{1}{2\sqrt{x}} + \frac{2}{x^5}$

8. $y = \pi^4$

$\cancel{\frac{dy}{dx} = 4\pi^3}$

$\boxed{\frac{dy}{dx} = 0}$

π^4 is constant
no change, therefore
derivative $= 0$.

F-L1a:

Evaluate each limit

9. $\lim_{x\to 3}\frac{2}{x+5}$ $= \frac{2}{3+5} = \frac{2}{8} = \frac{1}{4}$

10. $\lim_{x\to 2}\frac{2-x}{4-x^2}$ $= \frac{0}{0}$ ☹

$\lim_{x\to 2}\frac{\cancel{2-x}}{(\cancel{2-x})(2+x)}$

$\lim_{x\to 2}\frac{1}{2+x} = \frac{1}{2+2} = \frac{1}{4}$

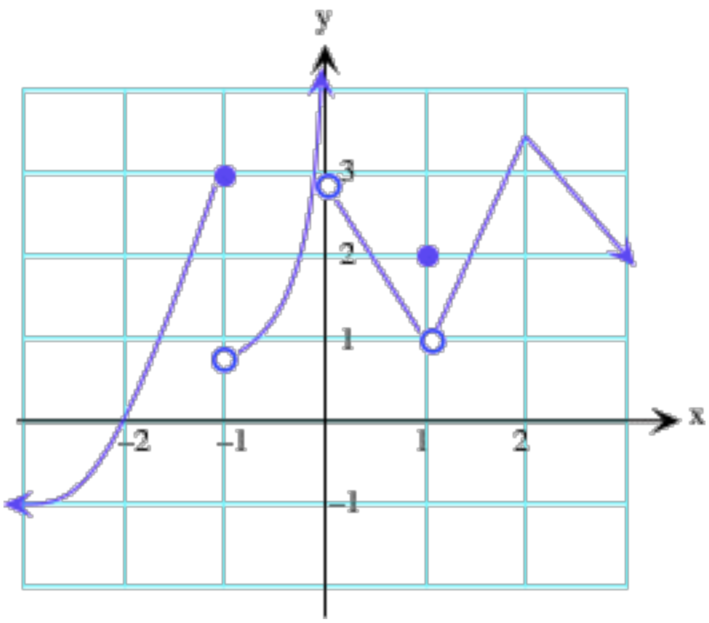
F-L2a:

Use the graph to rate each as true or false. If false, explain why.

11. $\lim_{x\to 0} f(x) = 3$

false. $\lim_{x\to 0^-} f(x) = \infty$ $\quad \infty \neq 3 \Rightarrow$ d.n.e.

$\lim_{x\to 0^+} f(x) = 3$



12. $\lim_{x\to 1} f(x) = 1$

true. the fact $f(1)=2$ is irrelevant here.

13. $\lim_{x\to -1^-} f(x) = 3$

true.

14. $\lim_{x\to -1} f(x) = 3$

false. $\lim_{x\to -1^-} f(x) = 3$

$\neq \rightarrow$ dne

$\lim_{x\to -1^+} f(x) = 0.75$

(estimate)

F-C3

15. Find and classify any discontinuities of the function. Justify your classifications using limits. $f(x) = \frac{x+5}{5-4x-x^2}$

$$f(x) = \frac{x+5}{5-4x-x^2} = \frac{x+5}{-1(x^2+4x-5)} = \frac{\cancel{x+5}}{-1(\cancel{x+5})(x-1)} = \frac{1}{-1(x-1)}$$

$x=-5$ is probably a removable discontinuity..

$x-1=0$

$x=1$ probably an infinite discontinuity.

$$\lim_{x\to -5} \frac{1}{-1(x-1)} = \frac{1}{-1(-5-1)} = \frac{1}{6}$$

$\lim_{x\to -5} f(x)$ exists, so f has a remov. disc. @ $x=-5$

$$\lim_{x\to 1} \frac{1}{-1(x-1)} = \frac{1}{0} \text{ ?? }$$ Need a one-sided limit.

$$\lim_{x\to 1^+} \frac{1}{-1(x-1)}$$

$$\frac{1}{-1(1^+-1)} = \frac{1}{-1(0^+)} = \frac{1}{0^-} = -\infty$$

because $\lim_{x\to 1^+} f(x) = -\infty$

f has an infinite discontinuity @ $x=1$.

F-B1:

16. Find any vertical and horizontal asymptotes of the function. Justify your answer with limits. $f(x) = \frac{2x-3}{x+5}$

$$f(x) = \frac{2x-3}{x+5}$$

V.a.

can't simplify or factor or whatever...
So when's the denom $=0$?

$x+5=0$

$x=-5$ justify

$$\lim_{x\to -5^-} \frac{2x-3}{x+5} = \frac{2(-5^-)-3}{-5^-+5} = \frac{-13^-}{0^-} = \infty$$

$\boxed{\text{V.a. @ } x=-5}$

h.a.

take $\lim_{x\to\infty} f(x)$

$$\lim_{x\to\infty} \frac{2x-3}{x+5} = \frac{2\infty \cancel{-3}}{\infty \cancel{+5}} = \frac{2\cancel{\infty}}{\cancel{\infty}} = 2$$

$\boxed{\text{h.a. @ } y=2}$