Solutions Nm

F-C4 Practice Assessment

Consider $f(x) = -2x^2 + 18x - 28$

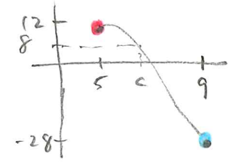
1. Is $f(x)$ continuous on the interval [5,9]? Explain.

yes. it's a polynomial, so it's continuous everywhere.

2. Explain why there must be a value c in [5,9] such that $f(c) = 8$ $\boxed{IVT}$

$f(5) = -2(5)^2 + 18(5) - 28 = 12$

$f(9) = -2(9)^2 + 18(9) - 28 = -28$



because $12 > 8 > -28$

$f(5) > f(c) > f(9)$ for some c in $[5,9]$.

3. Find the value or values of c in [5,9] such that $f(c) = 8$ guaranteed to exist in problem 2.

$f(c) = -2c^2 + 18c - 28 = 8$

$0 = 2c^2 - 18c + 36$

$0 = 2(c^2 - 9c + 18)$

$0 = 2(c-3)(c-6)$

$\cancel{c = 3}$ not in $[5,9]$ $\quad c = 6$

D-C1

4. Use the limit definition of derivative to show that if $f(x) = 2x^2 - 3x + 5$, then $f'(x) = 4x - 3$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 3(x+\Delta x) + 5 - (2x^2 - 3x + 5)}{\Delta x}$$

FOIL

$$\lim_{\Delta x \to 0} \frac{2(x^2 + 2x\Delta x + \Delta x^2) - 3x - 3\Delta x + 5 - 2x^2 + 3x - 5}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{2x^2} + 4x\Delta x + 2\Delta x^2 - \cancel{3x} - 3\Delta x + \cancel{5} - \cancel{2x^2} + \cancel{3x} - \cancel{5}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{4x\Delta x + 2\Delta x^2 - 3\Delta x}{\Delta x}$$

Factor out Δx

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(4x + 2\Delta x - 3)}{\cancel{\Delta x}} \Rightarrow \lim_{\Delta x \to 0} 4x + 2\Delta x - 3$$

$4x + 2(0) - 3 = 4x - 3$

D-C7

For each problem, find the derivative function.

Power Rule: $\frac{d}{dx} x^n = n x^{n-1}$

$\sqrt[a]{x^b} \rightleftarrows x^{\frac{b}{a}}$

$\frac{1}{x^n} \rightleftarrows x^{-n}$

5. $y = -3x^3 - 9x^5 + \frac{12}{5}x^4 + 2$ ready for calculus!

$y' = -3\cdot 3x^2 - 9\cdot 5x^4 + \frac{12}{5}\cdot 4x^3 + 0$

$\boxed{y' = -9x^2 - 45x^4 + \frac{48}{5}x^3}$

6. $f(x) = 4x^2 - \frac{4}{\sqrt[3]{x^2}} + 5\sqrt[5]{x^2}$

$f(x) = 4x^2 - 4\cdot\frac{1}{x^{2/3}} + 5x^{\frac{2}{5}}$

$f(x) = 4x^2 - 4x^{-2/3} + 5x^{2/5}$

calculus

$f'(x) = 4\cdot 2x^1 - 4\cdot -\frac{2}{3}x^{-\frac{5}{3}} + 5\cdot\frac{2}{5}x^{-\frac{3}{5}}$

$\boxed{f'(x) = 8x + \frac{8}{3}x^{-5/3} + 2x^{-3/5}}$

don't have to simplify for assessment.

if you simplify,

$\boxed{8x + \frac{8}{3\sqrt[3]{x^5}} + \frac{2}{\sqrt[5]{x^3}}}$

7. $y = \frac{2}{x} + \sqrt{x} - \frac{1}{2x^4}$

$y = 2x^{-1} + x^{\frac{1}{2}} - \frac{1}{2}\cdot\frac{1}{x^4}$

$y = 2x^{-1} + x^{\frac{1}{2}} - \frac{1}{2}x^{-4}$

$y' = 2\cdot -1x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}\cdot -4x^{-5}$

$\boxed{y' = -2x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + 2x^{-5}}$

if you simplify.

$\boxed{-\frac{2}{x^2} + \frac{1}{2\sqrt{x}} + \frac{2}{x^5}}$

8. $y = \pi^4 - \cos(x)$

π^4 is a constant!!

SHAME

$\cancel{4\pi^3}$

$y' = 0 - -\sin x$

$y' = \sin(x)$

$\frac{d}{dx}\sin(x) = \cos(x)$

$\frac{d}{dx}\cos(x) = -\sin(x)$

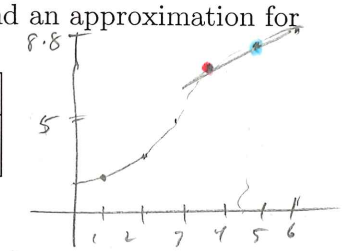
D-C6

Slope at 4.5

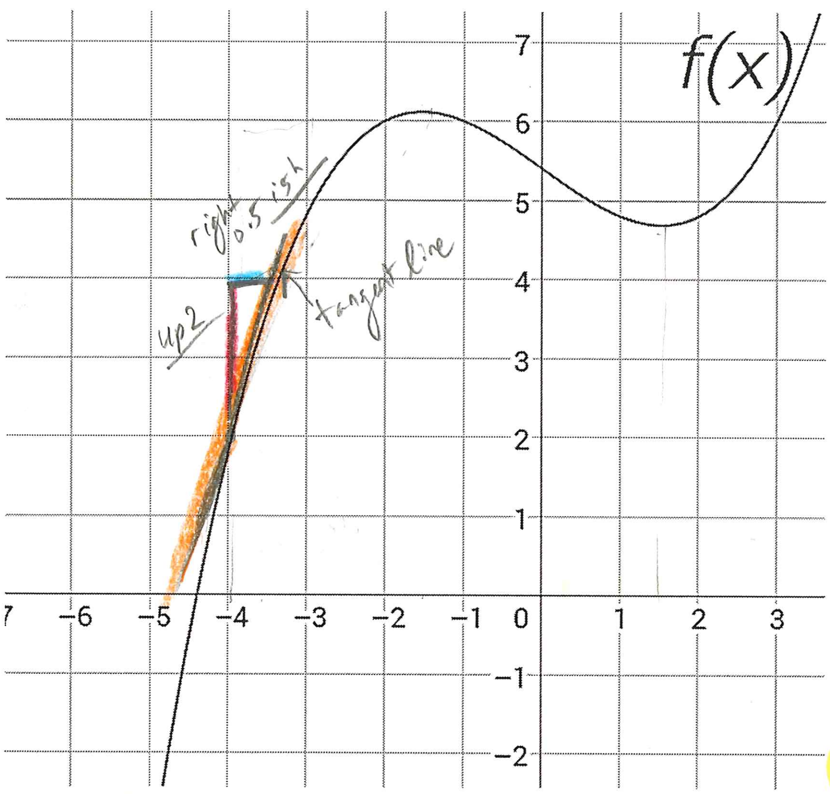
9. Below are selected values of $g(t)$, a differentiable function. From the data, find an approximation for the value of $g'(4.5)$.

4.5

t	1	2	3	4	5	6
$g(t)$	3.6	4.2	5.3	7.8	8.2	8.8



Approx slope @ x=4.5 ⇝ slope between (4, 7.8) ; (5, 8.2) ⇒ $\frac{y_2-y_1}{x_2-x_1} = \frac{8.2-7.8}{5-4} = 0.4$



10. Shown here is a graph of differentiable function $f(x)$. Find an approximation for the value of $f'(-4)$.

Approx. slope @ x = −4

Slope of tangent line appears to be

$\frac{\Delta y}{\Delta x} \approx \frac{+2}{+0.5} \approx 4$

11. Use the graph to approximate the value(s) of x where $f'(x) = 0$

peaks ; valleys

x = −1.5

x = 1.5

F-B1

12. Find the horizontal and vertical asymptotes, if any, of the function $f(x) = \frac{x+2}{x^3-x^2-6x}$. Justify your answers using limits.

H.A → $\lim_{x\to\infty} \frac{x+2}{x^3-x^2-6x} = \frac{\infty + 2}{\infty^3 - \cancel{\infty^2} - 6\infty} = \frac{\infty}{\infty^3} = \frac{\text{big}}{\text{ginormous!}} = 0$

⇒ $y = 0$ H.A

V.A Need to find candidates: Simplify, see what makes denominator = 0.

$f(x) = \frac{x+2}{x^3-x^2-6x} = \frac{x+2}{x(x^2-x-6)} = \frac{\cancel{x+2}}{x(\cancel{x+2})(x-3)} = \frac{1}{x(x-3)}$

Justify?

$x = 0$ $x = 3$ VA

$\lim_{x\to 0^+} \frac{1}{x(x-3)} = \frac{1}{0^+(0^+-3)} = \frac{1}{0^+(-3)} = \frac{1}{0^-} = -\infty$ ✓

$\lim_{x\to 3^+} \frac{1}{x(x-3)} = \frac{1}{3^+(3^+-3)} = \frac{1}{3^+(0^+)} = \frac{1}{0^+} = \infty$ ✓

F-C2

13. Find the values of a and b that make the function continuous everywhere. $f(x) = \begin{cases} -x+3, & x \le 0 \\ ax^2+b, & 0 < x \le 3 \\ 2x & x > 3 \end{cases}$

Cont @ x = 0?

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$-0+3 = -0+3 = a(0)^2+b$

$3 = \boxed{3 = b}$

Cont @ x = 3?

$\lim_{x\to 3^-} f(x) = f(3) = \lim_{x\to 3^+} f(x)$

$a(3)^2+b = a(3)^2+b = 2(3)$

$9a+b = 9a+b = 6$

$9a+3 = 6$

$9a = 3 \Rightarrow a = \frac{3}{9} = \frac{1}{3}$

F-C3

14. Find and classify any discontinuities of the function $y = \frac{1-x}{1-x^2}$. Justify your classifications using limits.

Simplify if possible $\Rightarrow x = 1$ removable?

$y = \frac{\cancel{1-x}}{\cancel{(1-x)}(1+x)} = \frac{1}{1+x}$ $\to$ $x = -1$ infinite?

$x = 1$

$\lim_{x\to 1} \frac{1}{1+x} = \frac{1}{1+1} = \frac{1}{2}$ ✓ $\boxed{x = 1 \text{ is removable disc.}}$

$x = -1$

$\lim_{x\to -1^-} \frac{1}{1+x} = \frac{1}{1+-1^-} = \frac{1}{1-1^-} = \frac{1}{0^-} = -\infty$ ✓ $\boxed{x = -1 \text{ is infinite disc.}}$

F-C1

15. Use $\boxed{\text{the definition of continuity}}$ to determine whether or not $f(x)$ is continuous everywhere.

$f(x) = \begin{cases} \sin(x), & x \ne \pi \\ 5, & x = \pi \end{cases}$

Continuous

continuous

left = middle = right ?

$\lim_{x\to \pi^-} f(x) = f(\pi) = \lim_{x\to \pi^+} f(x)$ ★ ← $\boxed{\text{must start with this.}}$

$\lim_{x\to \pi^-} \sin(x) = 5 = \lim_{x\to \pi^+} \sin(x)$

$\sin(\pi) = 5 = \sin(\pi)$

$0 \ne 5 \ne 0$

f(x) is continuous everywhere except $x = \pi$.