Implicit Differentiation, continued

$3\sin(y)=x^2$

$\frac{dy}{dx}\ ? = y'(x)$

"y is a function of x"

$\frac{d}{dx} 3\sin(y(x)) = \frac{d}{dx}x^2$

$3\cos(y(x))\,y'(x) = 2x$

$\frac{3\cos(y(x))\cdot y'(x)}{3\cos(y)} = \frac{2x}{3\cos(y)}$

$\boxed{y'(x) = \frac{2x}{3\cos(y)}}$

$\frac{d}{dx}g(f(x))$

$\frac{dg}{dx} = \frac{dg}{df}\cdot\frac{df}{dx}$

Good afternoon, no warm up, we will randomize then dive into this problem together :)

Find the slope of the line(s) tangent to $x^4+x^2y^3-y^2=1$ when $x=1$

$f: x^2 \quad g: y^3$

$f': 2x \quad g': 3y^2\cdot y'$

$$\frac{d}{dx}\left(x^4 + \overset{f}{\boxed{x^2}}\,\overset{g}{\boxed{y^3}} - y^2\right) = (1)\,\frac{d}{dx}$$

$$4x^3 + 2xy^3 + x^2\cdot 3y^2\cdot y' - 2y\cdot y' = 0$$

$$3x^2y^2\cdot y' - 2yy' = -4x^3 - 2xy^3$$

$$\frac{y'(3x^2y^2 - 2y)}{3x^2y^2-2y} = \frac{-4x^3-2xy^3}{3x^2y^2-2y}$$

$x=1$

what's y?

$1 + y^3 - y^2 = 1$

$y^2(y-1) = 0$

$y=0 \quad y=1$

$$y' = \frac{-4x^3 - 2xy^3}{3x^2y^2-2y}$$

$(1,0)$ $(1,1)$

$\frac{-4}{0}$ vertical tangent

$\frac{-6}{1}$

-6

Write the equation of the line(s) normal to $y^2 - x^3 = 2 - x$ where $x=0$

$(\perp)$

$y - y_1 = m(x - \cancel{x_1})$ with $x_1 = 0$

$y - y_1 = mx$

$y - \sqrt{2} = mx \Rightarrow y - \sqrt{2} = 2\sqrt{2}\,x$

$y + \sqrt{2} = mx \Rightarrow y + \sqrt{2} = -2\sqrt{2}\,x$

Finding y: $y^2 - 0 = 2 - 0$

$y^2 = 2$

$y = \pm\sqrt{2}$

$(0, \sqrt{2})$

$(0, -\sqrt{2})$

Find y':

$2y \cdot y' - 3x^2 = -1$

$y' = \dfrac{3x^2 - 1}{2y}$

$(0, \sqrt{2})$: $y' = -\dfrac{1}{2\sqrt{2}}$, $\perp$ $2\sqrt{2}$

$(0, -\sqrt{2})$: $y' = \dfrac{1}{2\sqrt{2}}$, $\perp$ $-2\sqrt{2}$

Show that the second derivative of y^2-xy=8 is $\dfrac{2y^2-2xy}{(2y-x)^3}$

$$\frac{d}{dx}(y^2 - xy) = (8)\frac{d}{dx}$$

$$2y\cdot y' - (1\cdot y + x\cdot y') = 0$$

$$2yy' - y - xy' = 0$$

$$y'(2y - x) = y$$

$$y' = \frac{y}{2y-x}$$

$$y' = \frac{y}{2y-x} \begin{matrix} \leftarrow f \\ \leftarrow g \end{matrix} \qquad \frac{f'g - fg'}{g^2}$$

f: y g: 2y − x

f': y' g': 2y' − 1

$$y'' = \frac{y'(2y-x) - y(2y'-1)}{(2y-x)^2}$$

$$\frac{\cancel{\frac{y}{2y-x}(2y-x)} - y\left(2\left(\frac{y}{2y-x}\right)-1\right)}{(2y-x)^2}$$

$$\frac{(2y-x)}{(2y-x)} \cdot \frac{\left(2y - \frac{2y^2}{2y-x}\right)}{(2y-x)^2}$$

$$\frac{2y(2y-x) - 2y^2}{(2y-x)^3} \rightarrow \frac{4y^2 - x - 2y^2}{(2y-x)^3} = \boxed{\frac{2y^2-2yx}{(2y-x)^3}}$$

Summary:
take implicit derivative, solve for y'
take implicit derivative OF y', using quotient rule
find y' in the answer, replace with the underlined red y'
simplify

Show that $y'' = \frac{2xy^2-x^4}{y^3}$ for $2x^3-3y^2=8$

to do later

Horizontal and Vertical Tangents

What does dy/dx tell you about y?

Slope $\frac{\text{change in } y}{\text{change in } x}$

~~ex~~

$$\frac{dy}{dx} = \frac{3x-2}{4} = 0 \Rightarrow x = \frac{2}{3}$$

When is y horizontal?

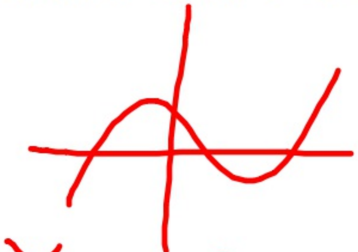
To find horizontal tangents: find derivative, find when numerator =0

To find vertical tangents: find derivative, find when denominator =0

Be sure to avoid numbers where BOTH =0 (as 0/0 is "indeterminate")

Find the location of the horizontal and vertical tangents of

$$y=\underbrace{(x-1)}_{f}\underbrace{(x^2-x-11)}_{g}$$



$$\frac{dy}{dx} = 1\cdot(x^2-x-11)+(x-1)(2x-1)$$

$$x^2-x-11+2x^2-3x+1$$

$$\frac{dy}{dx} = 3x^2-4x-10$$

$\boxed{\text{No v.t.}}$ $\boxed{\text{H.T?}}$

$$3x^2-4x-10=0$$

$$\cancel{(3x-10)(x+1)=0}$$

use quadratic formula to find when dy/dx =0

Find the location of the horizontal and vertical tangents of

$y=x\sqrt{4-x^2}$

(to do Friday)

Big ol' practice assessment

Will have time to work on it on Friday
Are able to do all skills right now except D-AD0 (will learn Friday)
Real thing is Monday

HW for Friday: p. 145 #3-39 (mult of 3)
(ignore parts about graphing or making it explicit)

Will have some time to
do retakes in class Frida
don't forget DS tomorrov
you should stay if D or F