

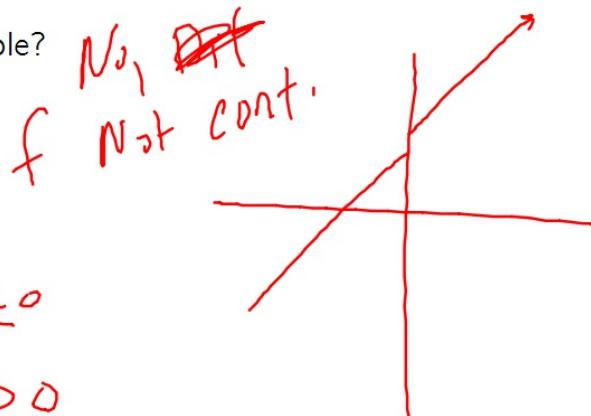
Warm up: do this in your head:

$$\lim_{h \rightarrow 0} \frac{\ln(2(x+h)) - \ln(2x)}{h}$$

derivative of this?

Review: Is the following function differentiable?

$$f(x) = \begin{cases} 3x + 5 & x \leq 0 \\ 3x + 4 & x > 0 \end{cases}$$



$$\text{Let } f(x) = \begin{cases} ax & x \leq 1 \\ bx^2 + x + 1 & x > 1. \end{cases}$$

$$f'(x) = \begin{cases} a & , x \leq 1 \\ 2bx + 1 & , x > 1 \end{cases}$$

Find a and b such that $f(x)$ is differentiable.

f cont?

$$f(1^-) = a \rightarrow a = b + 2$$

$$f(1^+) = b + 2 \rightarrow$$

$$\begin{aligned} & \begin{cases} a - b = 2 \\ a - 2b = 1 \end{cases} \\ & \underline{\underline{b = 1}} \end{aligned}$$

$$\begin{array}{c} \overline{\overline{a = 3}}} \\ \end{array}$$

f' cont?

$$f'(1^-) = a \rightarrow a = 2b + 1$$

$$f'(1^+) = 2b + 1 \rightarrow$$

<https://www.desmos.com/calculator/hbyvyz8ara>

Implicit Differentiation



Explicit Functions

$$y = -\frac{2}{3}x + 2$$

$$y = \sqrt{1-x^2}$$
$$y = (1-x^2)^{\frac{1}{2}}$$

Implicit Functions

$$2x + 3y = 6$$

$$x^2 + y^2 = 1$$

Why study this?

Many functions cannot be easily separated so that y is explicitly a function of x .

$$x^2 + xy = y$$
$$\frac{x^2}{1-x} = y$$
$$xy = x + y$$

$$xy - y = x$$

$$y(x-1) = x$$

$$y = \frac{x}{x-1}$$



But first...let's revisit the Chain Rule

$$\frac{dy}{dx} \ y = \frac{d}{dx} (\sin(x))^2$$

$$\frac{dy}{dx} = \frac{d}{d \sin(x)} (\sin(x))^2 \cdot \frac{d \sin(x)}{dx}$$

\downarrow \downarrow \downarrow

$$\frac{d}{d \sin(x)} \cdot \frac{d \sin(x)}{dx}$$

$$2 \sin(x) \cdot \cos(x)$$

How to do it:

math

$$\frac{d}{dx}(x^2 + xy + \cos(y)) = (8y) \frac{d}{dx}$$

procedure

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}\cos(y) = \frac{d}{dx}8y$$

$$2x + (1_y + x \cdot 1 \cdot \frac{dy}{dx}) + -\sin(y) \frac{dy}{dx} = 8 \frac{dy}{dx}$$

$$\underline{2x + y} + \underline{x \frac{dy}{dx}} - \underline{-\sin(y) \frac{dy}{dx}} = 8 \frac{dy}{dx}$$

$$x \frac{dy}{dx} - \sin(y) \frac{dy}{dx} - 8 \frac{dy}{dx} = -2x - y$$

$$\cancel{\frac{dy}{dx}}(x - \sin(y) - 8) = -2x - y$$

$$(x - \sin(y) - 8) \quad x - \sin(y) - 8$$

$$\frac{dy}{dx} = \frac{-2x - y}{x - \sin(y) - 8}$$

Things to remember:

- CHAIN RULE
- think of y as $y(x)$

Steps:

1. take derivative of both sides
2. use appropriate rules; take derivative of any non-x term as usual, but chain on a dy/dx on the end.
3. factor/solve for dy/dx . (answer usually has x's and y's in it.)

Graph:
<https://v>

Derivative of the Unit Circle Function

$$\frac{d}{dx}(x^2 + y^2) = (1) \frac{d}{dx}$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\cancel{2y} \frac{dy}{dx} = -\cancel{2x}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Another example:

$$x^3 + y^3 = xy \quad \frac{d}{dx}(x^3 + y^3) = (xy) \frac{d}{dx}$$

$$3x^2 + 3y^2 \frac{dy}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$$

$$3x^2 + 3y^2 \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$3y^2 \frac{dy}{dx} - x \frac{dy}{dx} = y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - x) = y - 3x^2$$

divide

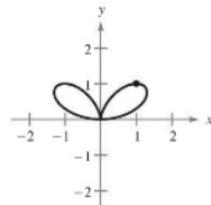
$$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$$

Finding Slope

31. Bifolium:

$$(x^2 + y^2)^2 = 4x^2y$$

Point: (1, 1)



$$\frac{d}{dx}[(x^2 + y^2)^2] = (4x^2 y) \frac{dy}{dx}$$

$$2(x^2 + y^2)^1(2x + 2y \frac{dy}{dx}) = 8xy + 4x^2 \frac{dy}{dx}$$

$$(2x^2 + 2y^2)(2x + 2y \frac{dy}{dx}) = 8xy + 4x^2 \frac{dy}{dx}$$

"distribute"

$$2x(2x^2 + 2y^2) + 2y(2x^2 + 2y^2) \frac{dy}{dx} = 8xy + 4x^2 \frac{dy}{dx}$$

$$4x^3 + 4xy^2 + (4x^2y + 4y^3) \frac{dy}{dx} = 8xy + 4x^2 \frac{dy}{dx}$$

$$(4x^2y + 4y^3) \frac{dy}{dx} - 4x^2 \frac{dy}{dx} = 8xy - 4x^3 - 4xy^2$$

$$\frac{dy}{dx}(4x^2y + 4y^3 - 4x^2) = 8xy - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} = \frac{8xy - 4x^3 - 4x^2y}{4x^2y + 4y^3 - 4x^2}$$

$$\frac{dy}{dx} = \frac{2xy - x^3 - x^2y}{x^2y + y^3 - x^2} .$$

plug in
point:
 $(1, 1)$
 $\uparrow 1$
 $x \quad y$

$$\left. \frac{dy}{dx} \right|_{(1,1)} = \frac{2 \cdot 1 \cdot 1 - (1)^3 - (1)^2 \cdot 1}{1^2 \cdot 1 + 1^3 - 1^2}$$

$$\frac{2 - 1 - 1}{1 + 1 - 1} = \frac{0}{1} = 0$$

\circlearrowleft
Whoaw!

Homework:

Calcchat.com

- AP MC Packet: (see website for solutions to some harder problems) [Due Fri]
 - Due Weds: P 393 # 10, 43, 60, 63, 72, 73 [D-AD2,3]
 - Due Fri: P 145 # 1, 3-30 (mult of 3) [D-AD5]
- Assessment Wednesday: log and inverse trig derivatives, writing normal line; chain rule, product rule quotient rule combined with all the known derivative rules