Good afternoon: warm up

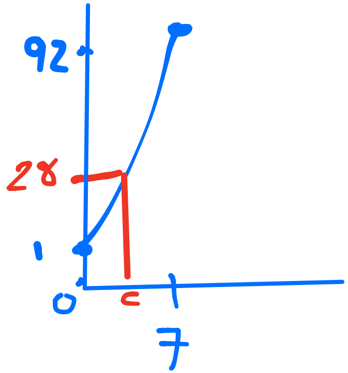
Find the value of c that is in [0,7] that is guaranteed to exist by the IVT for $f(x)=x^2+6x+1$ such that $f(c)=28$

f is cont. on $[0,7]$ b/c f is a polynomial. [Cont. everywhere]

Thus, IVT applies.

$f(0)=1 \qquad f(7)=7^2+6\cdot 7+1=92$

Because $1 \le 28 \le 92$, and there must be some c in $[0,7]$ $f(c)=28$.



$c^2+6c+1=28$

$c^2+6c-27=0$

$(c+9)(c-3)=0$

$\cancel{c=-9}$ Not in $[0,7]$ $\qquad c=3$

Assessments are being returned

Retakes available in DS Thursday and Friday

AP Limits Problems: Due Monday (no calculator)

Let's look at a few

$$\lim_{x\to 2^-} f(x) = f(2) = \lim_{x\to 2^+} f(x)$$

k

If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

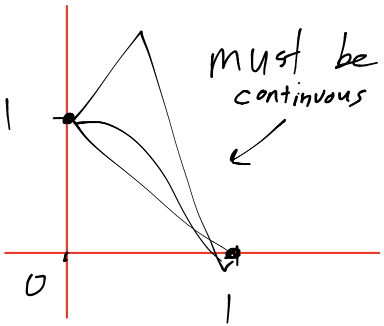
$$\lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = \frac{0}{0}$$

$$\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}} = \frac{2x+5-(x+7)}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$\frac{2x+5-x-7}{(x-2)(\sqrt{2x+5}+\sqrt{x+7})}$$

$$\frac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{2x+5}+\sqrt{x+7})}$$

$$\lim_{x\to 2} \frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{6}$$



must be continuous

Let g be a continuous function on the closed interval $[0,1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$. True, by E.V.T every cont. function has a maximum.

(B) For all a and b in $[0,1]$, if $a = b$, then $g(a) = g(b)$. True, vertical line test.

(C) There exists a number h in $[0,1]$ such that $g(h) = \frac{1}{2}$. True by I.V.T.

(D) There exists a number h in $[0,1]$ such that $g(h) = \frac{3}{2}$. Don't have to go thru 1.5.

(E) For all h in the open interval $(0,1)$, $\lim_{x\to h} g(x) = g(h)$.

True, def. of continuity

The paradox of dividing 0 by 0

$(x+h)(x+h)$

$$\lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 4 - (2x^2 - 3x + 4)}{h}$$

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 3(x+h) + 4 - 2x^2 + 3x - 4}{h}$$

$$\lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{3x} - 3h + \cancel{4} - \cancel{2x^2} + \cancel{3x} \cancel{-4}}{h}$$

$$\lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h}$$

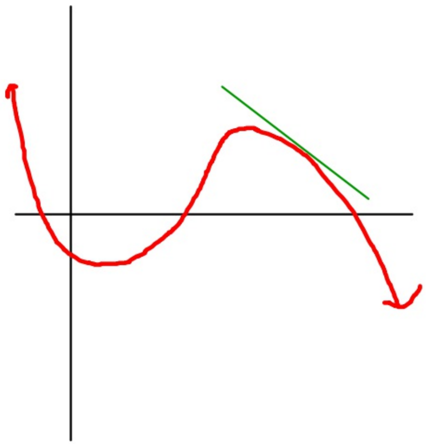
$$\lim_{h\to 0} \frac{\cancel{h}(4x+2h-3)}{\cancel{h}}$$

$$\lim_{h\to 0} 4x + 2h - 3$$

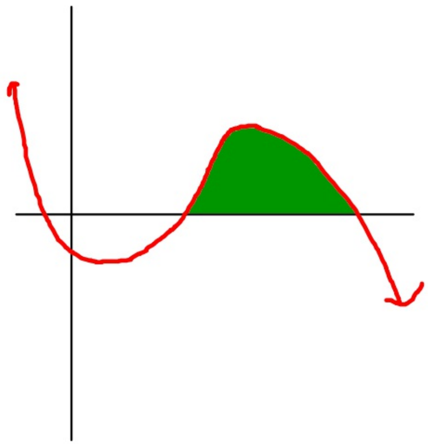
$4x + 2(0) - 3$

$= \boxed{4x-3}$

Two central questions in calculus

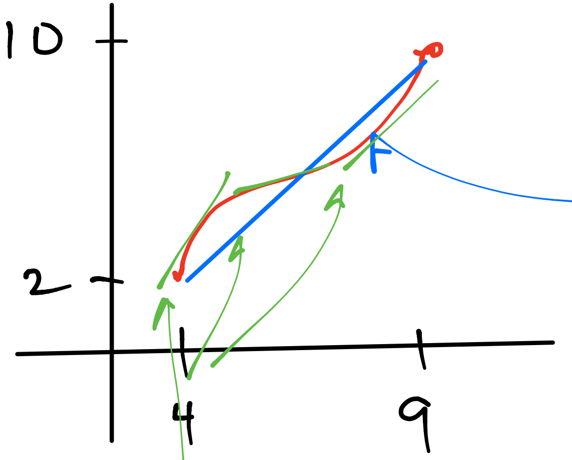


What is the slope of a curve
at a single point?



What is the exact area under a curve?

Average Rate of Change vs Instantaneous Rate of Change



Avg Rate of change over $[4,9] = \frac{8}{5}$ $\frac{\Delta y}{\Delta x}$

Slope of the secant line

Instantaneous Rates of change

???

Instant rate of change? How can change be measured 'instantly'?

https://www.youtube.com/watch?v=9vKqVkMQHKk

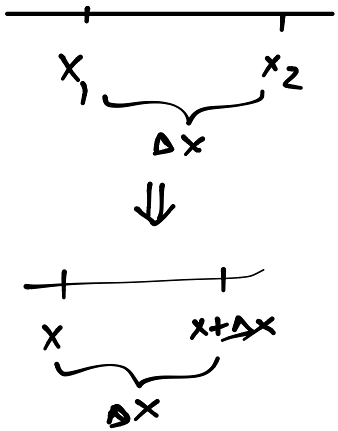
3:30 to 8:30

Slope Formula through the Ages

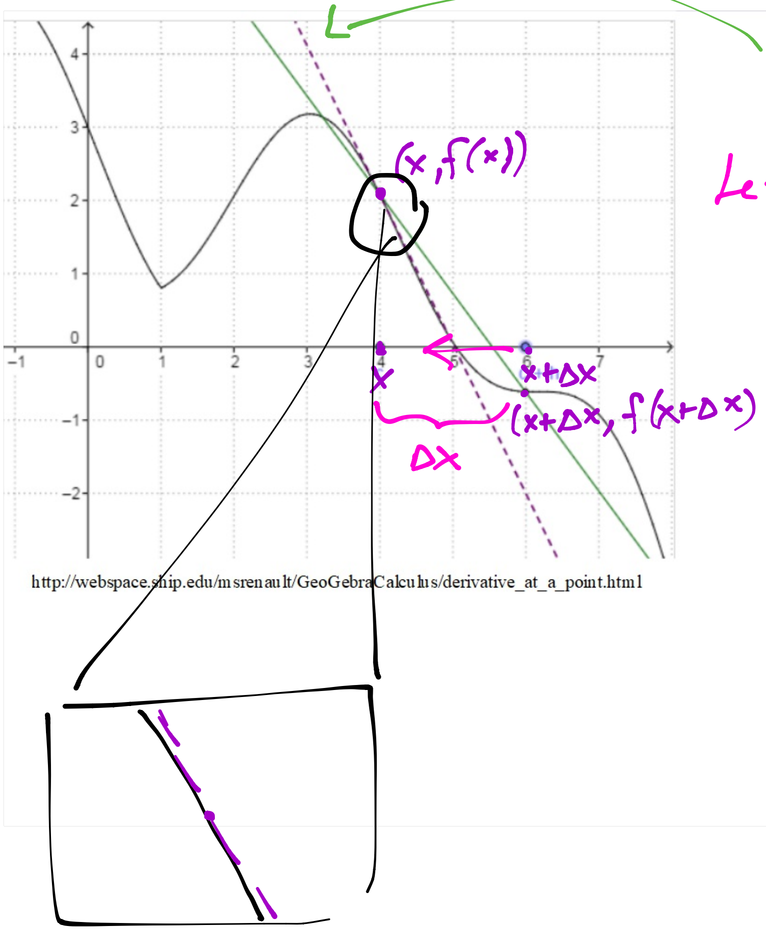
Algebra I: $\frac{\text{rise}}{\text{run}} \rightarrow \frac{y_2 - y_1}{x_2 - x_1}$

Function Notation: $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$

"Gap" $\frac{f(x+\Delta x) - f(x)}{\Delta x}$



Δx : size of "gap"



http://webspace.ship.edu/msrenault/GeoGebraCalculus/derivative_at_a_point.html

Slope of secant:

$$\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Let $\Delta x \to 0$. Now 2 pts converge to 1. Secant line becomes a TANGENT LINE.

How to do this?? TAKE A LIMIT!!

$$\underbrace{\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}}$$

this is the slope of the curve @ $(x, f(x))$

The Limit Definiton of Derivative

The slope of a line tangent to f(x) at (x,f(x)) is given by

$$\frac{df}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

"f prime"

Isaac Newton: $\dot{f}$

"the derivative of f with respect to x."

Gottfried Wilhelm Leibniz

(Sometimes h is used instead of Δx) [redacted]

HW p. 80 #87-90, 95-98 (use long division for 97)

AP Limits Packet due Monday