Good afternoon: warm ups

$y = \sqrt{\tan(2x^3)}$ Find $\frac{dy}{dx}$

$y = (\tan(2x^3))^{\frac{1}{2}}$

$y' = \frac{1}{2}(\tan(2x^3))^{-\frac{1}{2}} \cdot \sec^2(2x^3) \cdot 6x^2$

$y' = \frac{3x^2 \sec^2(2x^3)}{\sqrt{\tan(2x^3)}}$

$f(x) = 5x^2\csc(3x^2)$ Find $f'(x)$

$f: 5x^2$ $\quad g: \csc(3x^2)$

$f': 10x$ $\quad g': -\csc(3x^2)\cot(3x^2) \cdot 6x$

$10x\csc(3x^2) - 30x^3\csc(3x^2)\cot(3x^2)$

1) $\frac{dy}{dx} = 3(3x-1)^2 \cdot 3$

$= 9(3x-1)^2$

2) $\frac{dy}{dx} = \frac{1}{4}(4x^2+3)^{-\frac{3}{4}} \cdot 8x$

$= \frac{2x}{(4x^2+3)^{\frac{3}{4}}}$

3) $\frac{dy}{dx} = \frac{1}{2}(-4x^3+3)^{-\frac{1}{2}} \cdot -12x^2$

$= -\frac{6x^2}{(-4x^3+3)^{\frac{1}{2}}}$

4) $\frac{dy}{dx} = 4(-3x^4-4)^3 \cdot -12x^3$

$= -48x^3(-3x^4-4)^3$

5) $\frac{dy}{dx} = \frac{1}{5}(-3x-1)^{-\frac{4}{5}} \cdot -3$

$= -\frac{3}{5(-3x-1)^{\frac{4}{5}}}$

6) $\frac{dy}{dx} = 4(x^5+4)^3 \cdot 5x^4$

$= 20x^4(x^5+4)^3$

7) $\frac{dy}{dx} = \frac{1}{3}(5x^3+2)^{-\frac{2}{3}} \cdot 15x^2$

$= \frac{5x^2}{(5x^3+2)^{\frac{2}{3}}}$

8) $\frac{dy}{dx} = 5(-3x^4+2)^4 \cdot -12x^3$

$= -60x^3(-3x^4+2)^4$

9) $\frac{dy}{dx} = \frac{1}{4}(x^4-3)^{-\frac{3}{4}} \cdot 4x^3$

$= \frac{x^3}{(x^4-3)^{\frac{3}{4}}}$

10) $\frac{dy}{dx} = \frac{1}{4}(-3x^5-2)^{-\frac{3}{4}} \cdot -15x^4$

$= -\frac{15x^4}{\frac{3}{}}$

AP Prep:

If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

$f' = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}} \cdot (2x - 2)$

$f'(0) = \frac{2}{3}(-1)^{-\frac{1}{3}}(-2)$

$-\frac{4}{3} \quad \frac{1}{(-1)^{\frac{1}{3}}}$

-1

$\frac{4}{3}$ A

Your history with functions

- Constant
- Linear
- Absolute Value
- Quadratic
- Cubic, Quartic, Polynomial
- Rational
- Exponential
- Logarithmic
- Trigonometric
- Inverse Trigonometric

Can you take its derivative?

The Derivative of Log and Exponentials

Some review:

$x^b \, x^a = x^{b+a}$

$(x^b)^a = x^{ba}$

$\log_b a = x \rightleftharpoons b^x = a$

$\log_e x \rightleftharpoons \ln x$

$\log x + \log y = \log (xy)$

$\log x - \log y = \log \left(\frac{x}{y}\right)$

$\log b^a = a \log b$

$e \approx 2.718\ldots\ldots$

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\frac{d}{dx}\ln(x) = \lim_{\Delta x \to 0} \frac{\ln(x+\Delta x) - \ln(x)}{\Delta x}$$

$$\frac{1}{\Delta x}\ln\left(\frac{x+\Delta x}{x}\right)$$

$$\ln\left(\frac{x+\Delta x}{x}\right)^{\frac{1}{\Delta x}}$$

$$\ln\left(\frac{x}{x}+\frac{\Delta x}{x}\right)^{\frac{1}{\Delta x}}$$

$$\lim_{\Delta x \to 0} \ln\left(1+\frac{\Delta x}{x}\right)^{\frac{1}{\Delta x}}$$

Sub.

$$\frac{\Delta x}{x} = \frac{1}{n} \iff \Delta x = \frac{x}{n}$$

when $n \to \infty$, $\Delta x \to 0$

$$\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right)^{\frac{n}{x}}$$

$$\lim_{n\to\infty} \ln\left(\left(1+\frac{1}{n}\right)^{n}\right)^{\frac{1}{x}}$$

$$\lim_{n\to\infty} \frac{1}{x}\ln\left(1+\frac{1}{n}\right)^{n}$$

$$\frac{1}{x}\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right)^{n}$$

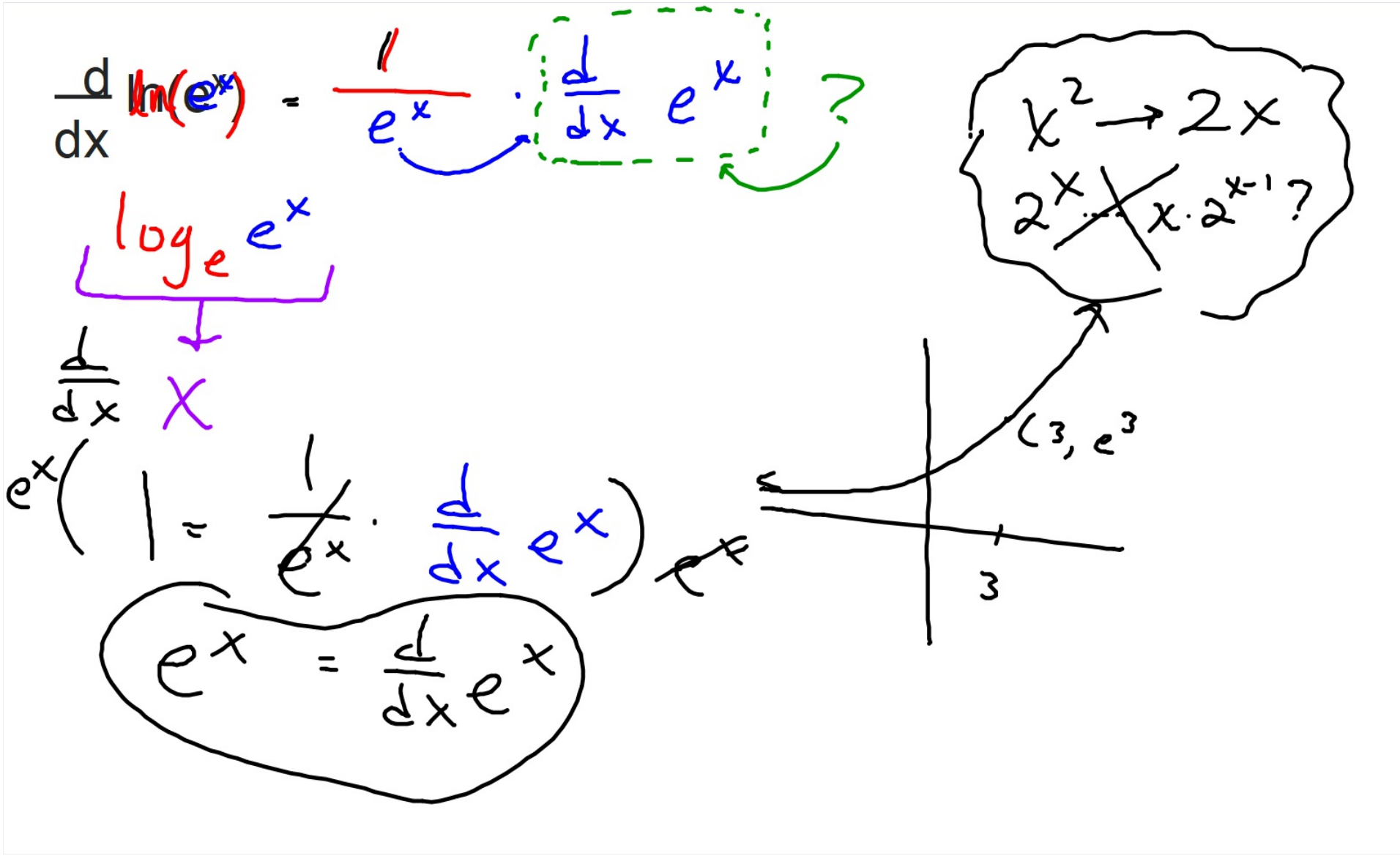
$$\frac{1}{x}\ln\left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\right]$$

$$\frac{1}{x}\ln e$$

$$\frac{1}{x}\cdot 1$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$(x^a)^b = x^{ab}$


$\frac{d}{dx}\ln(e^x) = \frac{1}{e^x} \cdot \frac{d}{dx} e^x$?
$x^2 \to 2x$
$2^x \quad x \cdot 2^{x-1}$?
$\log_e e^x$
$\frac{d}{dx}$ X
$e^x\left(1 = \frac{1}{e^x} \cdot \frac{d}{dx} e^x\right) e^x$
$e^x = \frac{d}{dx} e^x$
$(3, e^3)$
3

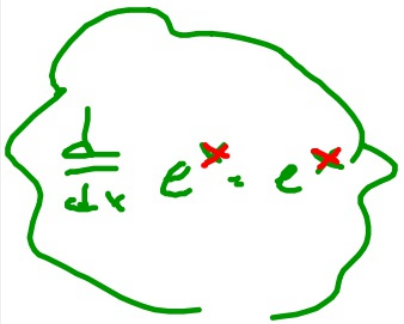
Exponential/Logarithmic Derivatives

(booklets)

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$\frac{d}{dx}a^x = a^x \ln\, a \qquad \frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

Find the derivative of $y=e^{\cos(x)}$



$y' = e^{\cos x} \cdot -\sin(x)$

$$-\sin(x) \cdot e^{\cos x}$$

Find the derivative of $y = \ln(5x^2-3)$

$$y' = \frac{1}{5x^2-3} \cdot 10x$$

$$\frac{10x}{5x^2-3}$$

$f(x)=x^2e^{2x}$

~~$f'(x)=2xe^{2x}$~~

$f: x^2 \quad g: e^{2x}$

$f': 2x \quad g': 2e^{2x}$

$2xe^{2x}+2x^2e^{2x}$

$$y = \frac{x+1}{\ln x} \quad \begin{matrix} \leftarrow f \\ \leftarrow g \end{matrix}$$

Quotient Rule

$f: x+1 \qquad g: \ln x$

$f': 1 \qquad g': \frac{1}{x}$

$$\Rightarrow \frac{f'g - fg'}{g^2}$$

$$\frac{1 \cdot \ln x - (x+1)\cdot \frac{1}{x}}{(\ln x)^2}$$

$$\leftarrow \frac{\ln x - \left(1 + \frac{1}{x}\right)}{(\ln x)^2}$$

$$x \cdot \frac{\ln x - 1 - \frac{1}{x}}{(\ln x)^2}$$

$$x \cdot$$

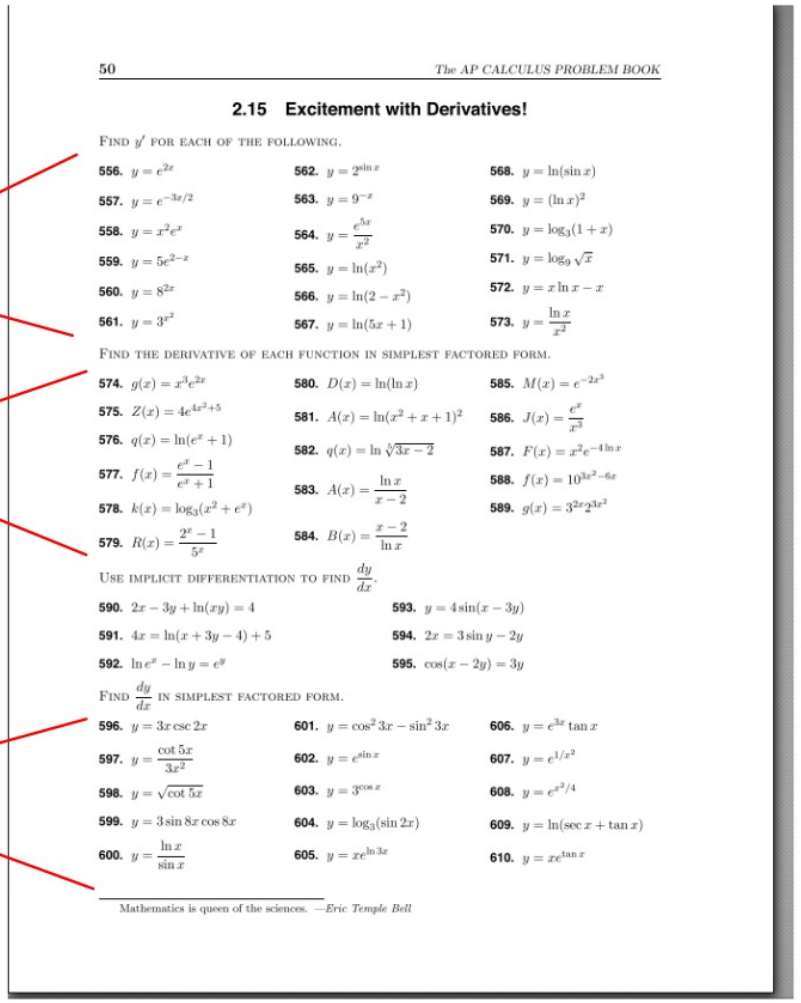
$$\frac{x \ln x - x - 1}{x(\ln x)^2}$$

HW

any 3

any 3

any 3



2.15 Excitement with Derivatives!

FIND y' FOR EACH OF THE FOLLOWING.

556. $y = e^{2x}$
557. $y = e^{-3x/2}$
558. $y = x^2e^x$
559. $y = 5e^{2-x}$
560. $y = 8^{2x}$
561. $y = 3^{x^2}$
562. $y = 2^{\sin x}$
563. $y = 9^{-x}$
564. $y = \dfrac{e^{5x}}{x^2}$
565. $y = \ln(x^2)$
566. $y = \ln(2 - x^2)$
567. $y = \ln(5x + 1)$
568. $y = \ln(\sin x)$
569. $y = (\ln x)^2$
570. $y = \log_3(1 + x)$
571. $y = \log_9 \sqrt{x}$
572. $y = x \ln x - x$
573. $y = \dfrac{\ln x}{x^2}$

FIND THE DERIVATIVE OF EACH FUNCTION IN SIMPLEST FACTORED FORM.

574. $g(x) = x^3e^{2x}$
575. $Z(x) = 4e^{4x^2+5}$
576. $q(x) = \ln(e^x + 1)$
577. $f(x) = \dfrac{e^x - 1}{e^x + 1}$
578. $k(x) = \log_3(x^2 + e^x)$
579. $R(x) = \dfrac{2^x - 1}{5^x}$
580. $D(x) = \ln(\ln x)$
581. $A(x) = \ln(x^2 + x + 1)^2$
582. $q(x) = \ln \sqrt[5]{3x - 2}$
583. $A(x) = \dfrac{\ln x}{x - 2}$
584. $B(x) = \dfrac{x - 2}{\ln x}$
585. $M(x) = e^{-2x^3}$
586. $J(x) = \dfrac{e^x}{x^3}$
587. $F(x) = x^2e^{-4\ln x}$
588. $f(x) = 10^{3x^2-6x}$
589. $g(x) = 3^{2x}2^{3x^2}$

USE IMPLICIT DIFFERENTIATION TO FIND $\dfrac{dy}{dx}$.

590. $2x - 3y + \ln(xy) = 4$
591. $4x = \ln(x + 3y - 4) + 5$
592. $\ln e^x - \ln y = e^y$
593. $y = 4\sin(x - 3y)$
594. $2x = 3\sin y - 2y$
595. $\cos(x - 2y) = 3y$

FIND $\dfrac{dy}{dx}$ IN SIMPLEST FACTORED FORM.

596. $y = 3x \csc 2x$
597. $y = \dfrac{\cot 5x}{3x^2}$
598. $y = \sqrt{\cot 5x}$
599. $y = 3\sin 8x \cos 8x$
600. $y = \dfrac{\ln x}{\sin x}$
601. $y = \cos^2 3x - \sin^2 3x$
602. $y = e^{\sin x}$
603. $y = 3^{\cos x}$
604. $y = \log_3(\sin 2x)$
605. $y = xe^{\ln 3x}$
606. $y = e^{3x} \tan x$
607. $y = e^{1/x^2}$
608. $y = e^{x^2/4}$
609. $y = \ln(\sec x + \tan x)$
610. $y = xe^{\tan x}$

Mathematics is queen of the sciences. —*Eric Temple Bell*