Mixed Review

505. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and at $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivatives of the following combinations of f and g at the given value of x.

a) $3f(x) - g(x), \quad x = -1$

b) $[f(x)]^3[g(x)]^3, \quad x = 0$

c) $g(f(x)), \quad x = -1$

d) $f(g(x)), \quad x = -1$

e) $\dfrac{f(x)}{g(x)+2}, \quad x = 0$

f) $g(x + f(x)), \quad x = 0$

521 (AP, 2000AB). Consider the curve given by $xy^2 - x^3y = 6$.

a) Find $\dfrac{dy}{dx}$.

b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

c) Find the x-coordinate of each point on the curve where the tangent is vertical.

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\dfrac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

FIND $\dfrac{dy}{dx}$ IN SIMPLEST FACTORED FORM.

596. $y = 3x \csc 2x$

597. $y = \dfrac{\cot 5x}{3x^2}$

598. $y = \sqrt{\cot 5x}$

599. $y = 3 \sin 8x \cos 8x$

600. $y = \dfrac{\ln x}{\sin x}$

601. $y = \cos^2 3x - \sin^2 3x$

602. $y = e^{\sin x}$

603. $y = 3^{\cos x}$

604. $y = \log_3(\sin 2x)$

605. $y = xe^{\ln 3x}$

606. $y = e^{3x} \tan x$

607. $y = e^{1/x^2}$

608. $y = e^{x^2/4}$

609. $y = \ln(\sec x + \tan x)$

610. $y = xe^{\tan x}$

525. Find the volume and surface area of a cube of side length 6.

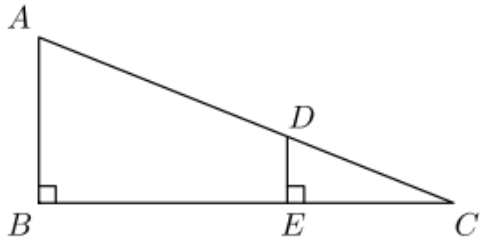
526. Find the volume and surface area of a box with dimensions 3, 4, and 5.

527. What is the hypotenuse of a right triangle with legs 5 and 12?

528. The area of an isosceles right triangle is 8. What is the length of its hypotenuse?

529. A cylinder is constructed so that its height is exactly 4 times its radius. If the volume of the cylinder is 500π, then what is its radius?

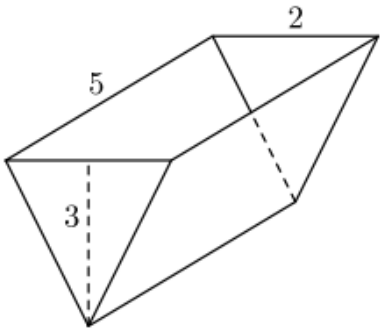
530. In the figure to the right, $DE = 2$, $EC = 5$, and $AB = 5$. Find the lengths of $\overline{AC}$ and $\overline{BC}$.



531. What is the area of an equilateral triangle if its side lengths are 8?

532. What is the area of a semicircle of radius 10?

533. The trough shown in the figure at the right is 5 feet long and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Find the volume of water in the trough when the trough is full.



534. A cone is constructed so that its height is exactly 4 times its radius. If the volume of the cone is 324π, then what is its radius?

535. A 12-foot ladder is leaning against a wall so that it makes a 60° angle with the ground. How high up the wall does the ladder touch the wall?

536. An equilateral triangle has an area of $4\sqrt{3}$. What is the height of this equilateral triangle?