Good afternoon: warm up in notebooks

We will assess at the latter part of class

1. Write the equation of the line tangent to $y = \frac{1}{2}\sin(x)$ at the point where $x = \frac{\pi}{2}$

$$f' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. Find the limit: $\lim_{\Delta x \to 0} \frac{-\cos(x+\Delta x) - -\cos(x)}{\Delta x}$

$f' =$

$f = -\cos(x)$

$f' = \sin(x)$

Reminders:

quarter ends in 9 school days

retakes available in Mon/Tues DS

tutoring/retakes Tues 4-5p

$y = \frac{1}{2}\sin(x)$ $\quad x = \frac{\pi}{2}$

$y - y_1 = m(x - x_1) \Rightarrow y - \frac{1}{2} = 0\left(x - \frac{\pi}{2}\right)$

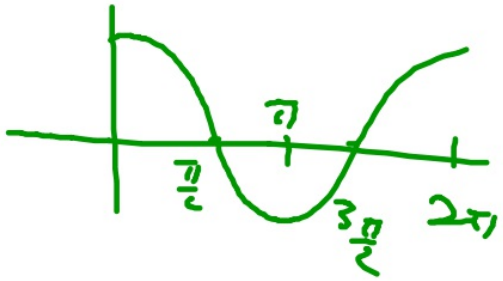
$y = \frac{1}{2}$

- Finding y_1: $y\left(\frac{\pi}{2}\right) = \frac{1}{2}\sin\left(\frac{\pi}{2}\right) = \frac{1}{2}$ $\quad \left(\frac{\pi}{2}, \frac{1}{2}\right)$

- Finding m: $\frac{dy}{dx} = \frac{1}{2}\cos(x)$

$$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = \frac{1}{2}\cos\left(\frac{\pi}{2}\right)$$

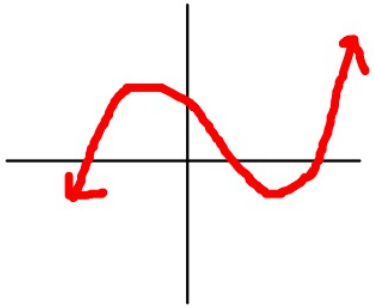
$$f'\left(\frac{\pi}{2}\right) = 0$$



Visibly Random Grouping

Which one doesn't belong? Be prepared to defend your selection

A



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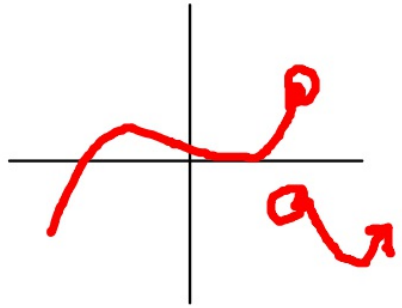

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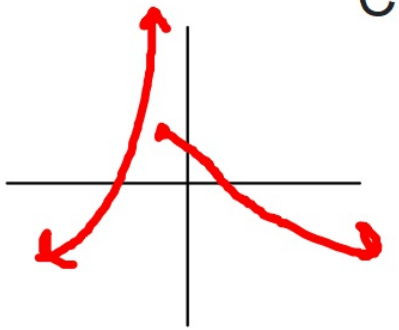
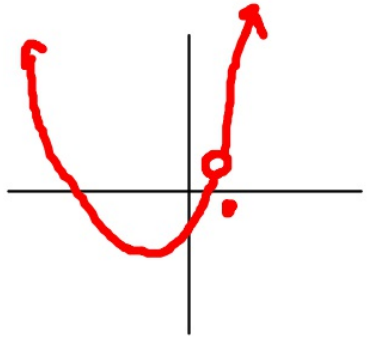


D

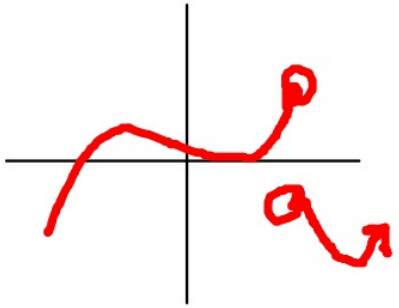


Go-around protocol: one person at a time shares reasoning until all 4 have spoken

Find a reason why each one might not belong

A

B



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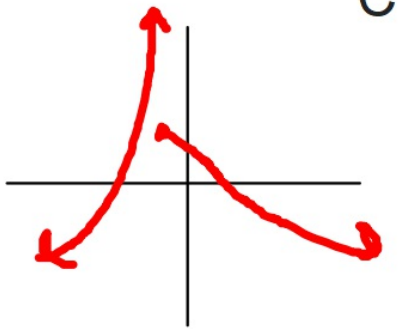
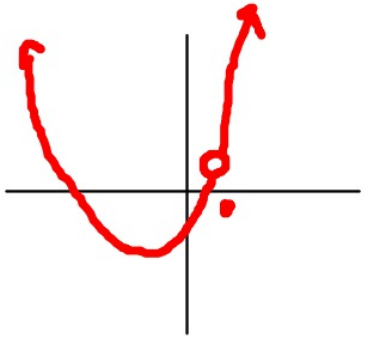
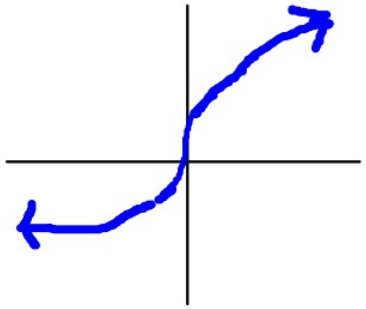


D



Which one doesn't belong?

A



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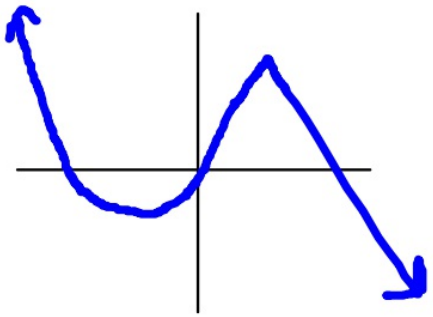

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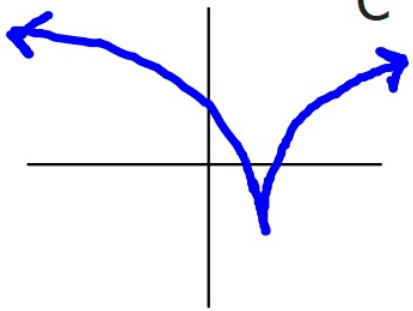

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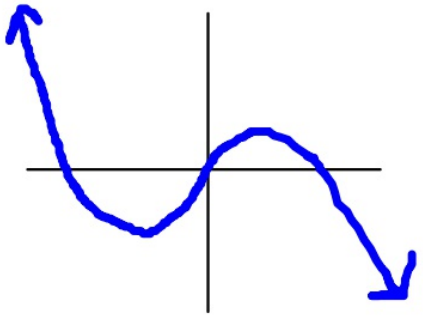

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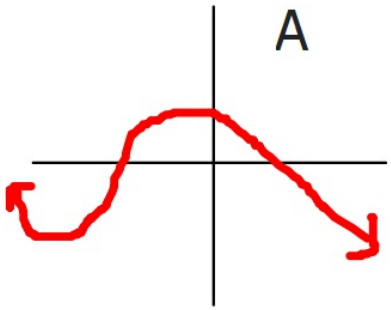
A



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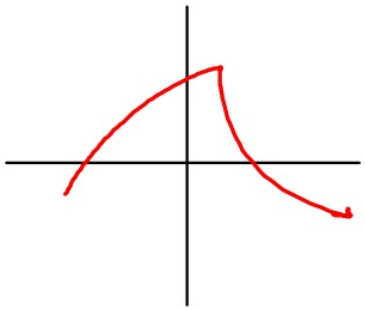

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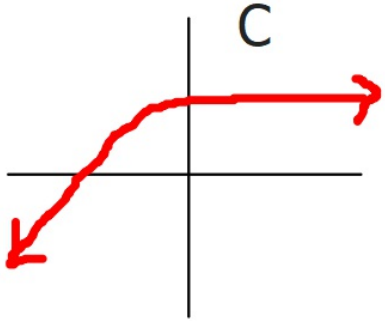

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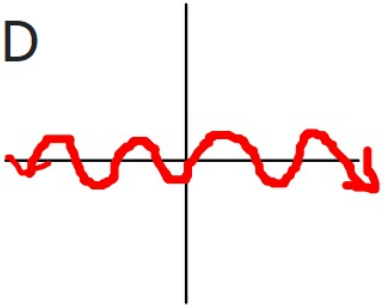


Some terminology

Derivative (at a point): slope of the tangent line (a number)

Derivative (as a function): a separate function whose output is the original's slope

Differentiation: the process of finding a derivative (verb: differentiate)

"Derivative with respect to ___": identifying the independent variable of the change (usually x, sometimes t)

$$\frac{d}{dx}$$

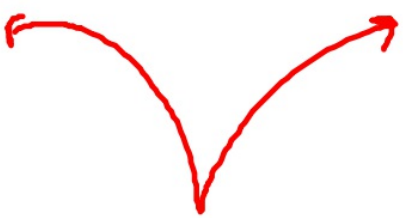
Differentiability through 3 lenses:

Verbally: A continuous function is differentiable where
there exists a unique tangent line with a defined slope

Graphically: - continuous (no asymptotes, holes, jumps)
- "smooth" (no cusps, corners, vertical tangents)

Algebraically: f(x) is continuous
f '(x) is also continuous

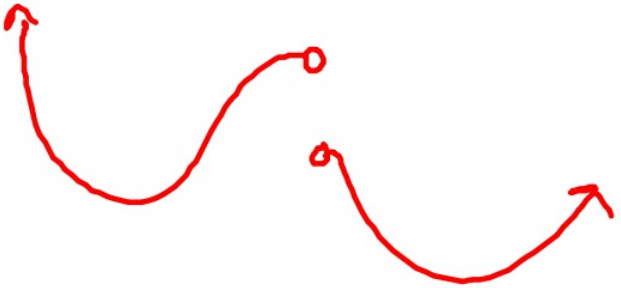
Types of non-differentiable points



Cusp

Corner

Vertical tangent



Discontinuity

take a few minutes to study over it, discuss with neighbors

Optional Practice

F-C4: p.81: #95-97

D-C1: p.103: #9, 10, 17, 18

D-C7: p114: #39-52

F-B1: p92: #68-70

F-C2: p80: #61-65

F-C3: p80: 39-48

F-C1: p.80 #75-76

Linear Approximations

slope

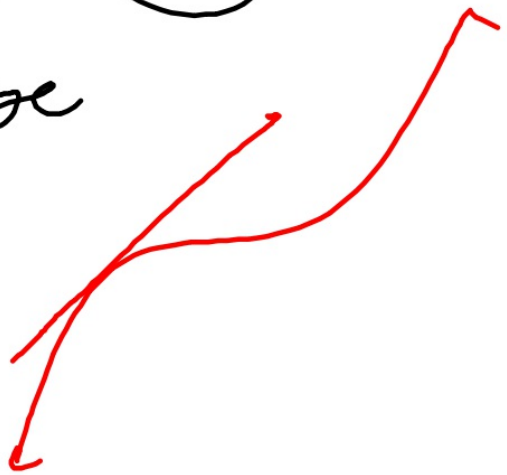
- Find the avg rate of change over $[3, 5]$ of $f(x) = x^2 - 2x + 1$.

$$\frac{f(5) - f(3)}{5 - 3} = \frac{16 - 4}{2} = \frac{12}{2} = 6$$

- Find the instant. rate of change $f(x) = x^2 - 2x + 1$ @ $x = 3.5$

$f'(x) = 2x - 2$

$f'(3.5) = 2(3.5) - 2 = 5$



Use the linear approximation of $y = \sqrt{x}$ to approximate $\sqrt{9.2}$

1. Need to write the tangent line...but using what point??
   note that (using common sense) that $\sqrt{9}$ is 3, so this function passes through the point (9,3)

$$y - y_1 = m(x - x_1)$$

$$y - 3 - m(x - 9)$$

2. Need the slope...so take the derivative

$$y = \sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$$

3. Plug in x=9 to find slope at the point (9,3)

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$$ this is m

4. So tangent line is:

$$y - 3 = \frac{1}{6}(x - 9)$$

Plug in the weird number (9.2) and solve for y...this is approx. $\sqrt{9.2}$

$$y - 3 = \frac{1}{6}(9.2 - 9)$$

$$y - 3 = \frac{1}{6}\left(\frac{2}{10}\right)$$

$$y - 3 = \frac{1}{30} \longrightarrow \boxed{y = 3\frac{1}{30}}$$