Solutions

D-AD17

Practice Assessment

1. The position, in feet, of a particle moving along a straight path is given by the differentiable function $x(t) = \sin 2t - \cos 4t$ where t is measured in seconds. Find the acceleration of the particle at t=0. Include units in your answer.

pos → v → acc
f   f′   f″

$x(t) = \sin 2t - \cos 4t$

$x'(t) = v(t) = \cos 2t \cdot 2 - -\sin 4t \cdot 4$

$= 2\cos 2t + 4\sin 4t$

$x''(t) = v'(t) = a(t) = -2\sin 2t \cdot 2 + 4\cos 4t \cdot 4$

$a(t) = -4\sin 2t + 16\cos 4t$

$a(0) = -4\sin(2\cdot 0) + 16\cos(4\cdot 0)$

$= 16\ ft/s^2$

2. The position, in feet, of a particle moving along a straight path is given by the differentiable function $s(t) = -\cancel{x}t^3 + 5\cancel{x}t^2 - 7\cancel{x}t + 3$. Find all times t where the particle is at rest.

Let $x = t$

means velocity = 0 → $s'(t)$

$s = -x^3 + 5x^2 - 7x + 3$

$\frac{ds}{dt} = -3x^2 + 10x - 7 = 0$ ← "at rest"

$-1(3x^2 - 10x + 7) = 0$

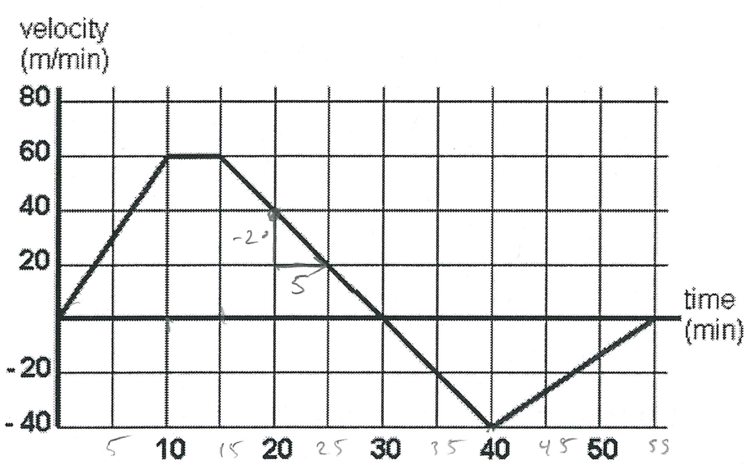
$-1(3x - 7)(x - 1) = 0$

$x = 7/3$ $\quad x = 1$

$t = 7/3$ sec

$t = 1$

3. A person is hiking in a national park and their velocity in meters per minute is graphed below.



When does the person change direction?

"velocity changes sign"

$t = 30$ min

When is the person walking at her greatest speed? → $|v(t)|$

$t = (10, 15)$ or, from 10 min to 15 min.

When is the person slowing down?

"$v(t)$ approaches 0"

$(15, 30)$ interval and $(40, 55)$

Calculate the acceleration at t=20min.

$v'(t)$

slope of velocity

$a(t) = v'(t) = \frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{-20\ m/min}{5\ min} = -4\ m/min^2$

D-AD5

4. Find the slope of the tangent line at $x = -1$ when $y = xy + x^2 + 1$

(1) Point? plug $x=-1$ to find y. $y = -1y + (-1)^2 + 1$ $(-1,1)$

$2y = 2 \rightarrow y = 1$

(2) Slope? $\frac{d}{dx}\left[y\right] = \left[\underbrace{xy}_{fg} + x^2 + 1\right]\frac{d}{dx}$

$\frac{dy}{dx} = \underbrace{1 \cdot y + x \cdot \frac{dy}{dx}}_{\text{product rule}} + 2x + 0$ subtract

$\frac{dy}{dx} - x \cdot \frac{dy}{dx} = y + 2x$

factor $\frac{dy}{dx}(1-x) = y + 2x$

$\frac{dy}{dx}\frac{(1-x)}{1-x} = \frac{y+2x}{1-x}$

$\frac{dy}{dx} = \frac{y+2x}{1-x}$

(3) plug in $(-1,1)$

$\frac{1+2(-1)}{1--1} = \frac{1-2}{1+1} = -\frac{1}{2}$

5. If $x^2 + xy + y^3 = 0$, then find $\frac{dy}{dx}$

$\frac{d}{dx}\left[x^2 + \underbrace{xy}_{fg} + y^3 = 0\right]\frac{d}{dx}$

$2x + \underbrace{1 \cdot y + x \cdot \frac{dy}{dx}}_{\text{prod rule}} + 3y^2\frac{dy}{dx} = 0$ chain Rule

$x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -2x - 1y$

$\frac{dy}{dx}(x + 3y^2) = -2x - 1y \Longrightarrow \boxed{\frac{dy}{dx} = \frac{-2x-y}{x+3y^2}}$

D-CD7

6. Write the equation of the line tangent to $y = (2x-1)^4$ where x=1

(1) point: $y = (2(1)-1)^4 \rightarrow (2-1)^4 \rightarrow (1)^4 = 1$ $(1,1)$ $x_1\ y_1$

(2) Slope: $\frac{dy}{dx} = 4(2x-1)^3 \cdot 2 \rightarrow 8(2x-1)^3$

plug in $x=1$

$\rightarrow 8(1)^3 \rightarrow 8 \leftarrow m$

(3) $y - y_1 = m(x - x_1)$

$\boxed{y - 1 = 8(x-1)}$

or

$y = 8x - 7$

7. Write the equation of a line with a slope of 6 that is tangent to $y = x^2 - 4x + 3$.

Given Slope!
Need point. $\longrightarrow$ $\frac{dy}{dx} = 6 \longrightarrow \frac{dy}{dx} = 2x - 4 = 6$

$2x = 10$

$x = 5$

$y = x^2 - 4x + 3$

$= 5^2 - 4 \cdot 5 + 3$

$25 - 20 + 3$

$= 8 \rightarrow (5,8) \rightarrow \boxed{y - 8 = 6(x-5)}$

D-CD4

8. Show that $f(x) = \begin{cases} 5x^2 - 3x - 6 & x \le 1 \\ -2x^2 - 2 & x > 1 \end{cases}$ is not differentiable at $x = 1$.

$x \to 1^-$

$x \to 1^+$

$f'(x) = \begin{cases} 10x - 3, & x \le 1 \\ -4x, & x > 1 \end{cases}$

Is f continuous @ x=1?

$\lim_{x\to1^-} f(x) = f(1) = \lim_{x\to1^+} f(x)$

$-4 = -4 = -4$

✓ yes

Is f'(x) continuous @ x=1?

$\lim_{x\to1^-} f'(x) = f'(1) = \lim_{x\to1^+} f'(x)$

$10(1) - 3 = 10(1) - 3 = -4(1)$

$7 = 7 \ne -4$

NO

9. Find the values of *a* and *b* that would make *f(x)* differentiable. $f(x) = \begin{cases} ax^2 + bx - 2 & x \le 2 \\ -2x^2 + 2x + 8 & x > 2 \end{cases}$

f cont?

plug 2 into both f functions, set =

$4a + 2b - 2 = -2(4) + (4) + 8$

$4a + 2b - 2 = 4$

$4a + 2b = 6$

$f'(x) = \begin{cases} 2ax + b, & x \le 2 \\ -4x + 2, & x > 2 \end{cases}$

f' cont?

plug 2 into both f' functions, set =

$4a + b = -8 + 2$

$4a + b = -6$

Solve system

$\begin{cases} 4a + 2b = 6 \\ -(4a + b = -6) \end{cases} \Longrightarrow$ Elimination $b = 12$

$0 + 1b = 12$

plug back in

$4a + 2(12) = 6$

$4a + 24 = 6$

$4a = -18$

$a = -9/2$

D-AD18

10. Use a tangent line to approximate the value of $\sqrt[3]{29}$

$y = \sqrt[3]{x} = x^{1/3}$

$y' = \frac{1}{3} x^{-2/3}$

$y' = \frac{1}{3} \cdot \frac{1}{x^{2/3}}$

$y' = \frac{1}{3(\sqrt[3]{x})^2}$

$(27, 3)$

$y'(27) = \frac{1}{3(\sqrt[3]{27})^2} = \frac{1}{3(3)^2}$

$y'(27) = \frac{1}{27}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{27}(x - 27)$

plug in x = 29

$y - 3 = \frac{1}{27}(29 - 27)$

$y - 3 = \frac{1}{27}(2)$

$\boxed{y = 3\frac{2}{27}}$

11. Use a tangent line to approximate the value of $\cos\frac{\pi}{5}$ [Note: $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$]

$y - y_1 = m(x - x_1)$

$y - \frac{\sqrt{3}}{2} = m\left(x - \frac{\pi}{6}\right)$

m @ x = π/6 ?

$y = \cos x$

$y' = -\sin x$

$y'(\pi/6) = -\sin(\pi/6)$

$= -\frac{1}{2}$

$x_1 \quad y_1$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(x - \frac{\pi}{6}\right)$

plug in x = π/5

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{\pi}{5} - \frac{\pi}{6}\right)$

$y - \frac{\sqrt{3}}{2} = -\frac{1}{2}\left(\frac{\pi}{30}\right)$

$y = \frac{\sqrt{3}}{2} - \frac{\pi}{60} \approx \boxed{0.814}$

numerator $= 0$ denominator $= 0$

D-CD5

12. Find the x-values of any horizontal and vertical tangents to $f(x) = x^4 - 8x^2 + 2$.

$$\frac{dy}{dx} = \frac{4x^3 - 16x}{1} = 0$$

V.T. No v.t.

H.T.

$4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x(x+2)(x-2) = 0 \Rightarrow x = 0, 2, -2$ H.T.

13. Find the x-values of any horizontal and vertical tangents to $y = 2\sqrt{x} - \frac{1}{2}x^2$

$y = 2x^{\frac{1}{2}} - \frac{1}{2}x^2$

$\frac{dy}{dx} = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} \cdot 2x$

$\frac{dy}{dx} = x^{-\frac{1}{2}} - x \leftarrow$ Factor "out" $x^{-1/2}$

$\frac{dy}{dx} = x^{-\frac{1}{2}}(1 - x^{1})$ 3/2 (sorry)

$\frac{dy}{dx} = \frac{1 - x}{\sqrt{x}}$ → H.T. $1 - x = 0 \Rightarrow x = 1$ H.T.

→ V.T. $\sqrt{x} = 0 \Rightarrow x = 0$ V.T.

D-AD0

Evaluate each limit using L'hopital's Rule.

14. $\lim_{\theta \to 0} \frac{\sin 8\theta}{6\theta}$ direct sub $\frac{0}{0}$:-|

$\lim_{\theta \to 0} \frac{8\cos 8\theta}{6} \Rightarrow \frac{8\cos 0}{6} = \frac{8}{6} = \frac{4}{3}$

15. $\lim_{x \to 0} \frac{e^{3x} - 2^x}{3x}$ direct sub $= \frac{e^0 - 2^0}{3(0)} = \frac{0}{0}$:-|

$\lim_{x \to 0} \frac{3e^{3x} - 2^x \ln 2}{3} \Rightarrow \frac{3e^0 - 2^0 \ln 2}{3} \Rightarrow \frac{3 - \ln 2}{3}$

16. $\lim_{x \to \infty} x^2 e^{-x}$

$\lim_{x \to \infty} \frac{x^2}{e^x}$ direct sub $= \frac{\infty^2}{e^\infty} = \frac{\infty}{\infty}$:-|

l'hop

$\lim_{x \to \infty} \frac{2x}{e^x} = \frac{2\infty}{\infty}$:-|

l'hop

$\lim_{x \to \infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$