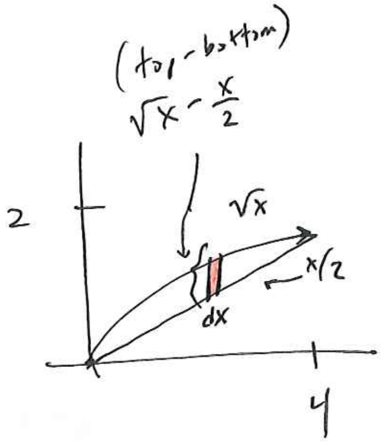
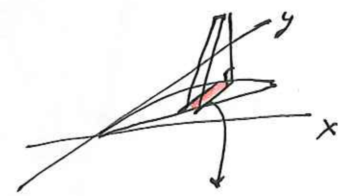
#1.

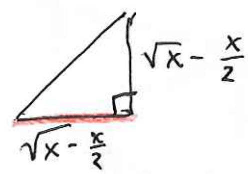
(1) Graph Equations / Base Region;
Find Intersections if Needed.



(2) Draw representative Rectangle,
3D view and 2D view
of cross section



(3) Find Area of Cross Section
$A(x)$ in terms of x



$$V = \int_a^b \underbrace{A(x)}_{\text{Face Area of Cross-Section}} \, dx$$

$$A(x) = \frac{1}{2} b \cdot h = \frac{1}{2}\left(\sqrt{x} - \frac{x}{2}\right)\left(\sqrt{x} - \frac{x}{2}\right)$$

"Foil", etc.

$$A(x) = \frac{1}{2}\left(x - x^{3/2} + \frac{1}{4}x^2\right)$$

$$V = \int_0^4 \frac{1}{2}\left(x - x^{3/2} + \frac{1}{4}x^2\right) dx$$

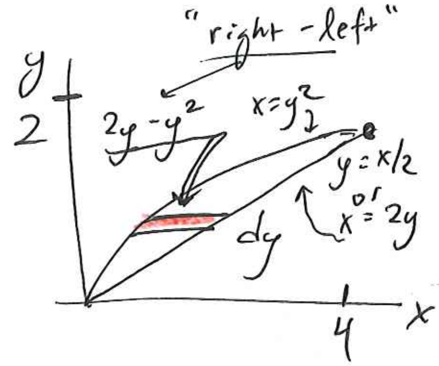
$$V = \frac{1}{2}\int_0^4 x - x^{3/2} + \frac{1}{4}x^2 \, dx$$

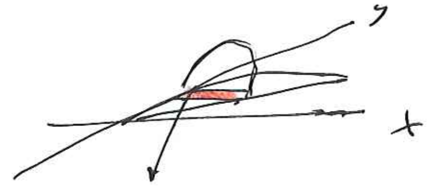
Calc. (Not hard by hand)

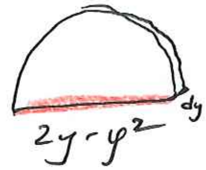
$$= \frac{1}{2}\left(\frac{8}{15}\right)$$

$$= \boxed{\frac{8}{30} \text{ cubic units}}$$

#2.







Diameter: $2y - y^2$

Radius: $y - \frac{1}{2}y^2$

$$A(y) = \frac{1}{2} \cdot \pi r^2$$

$$A(y) = \frac{\pi}{2} \cdot \left(y - \frac{1}{2}y^2\right)^2$$

$$V = \int_0^2 \frac{\pi}{2}\left(y - \frac{1}{2}y^2\right)^2 dy$$

$$= \frac{\pi}{2} \underbrace{\int_0^2 \left(y - \frac{1}{2}y^2\right)^2 dy}_{\text{CALC.}}$$

$$= \frac{\pi}{2} \cdot \frac{4}{15}$$

$$= \frac{4\pi}{30} \longrightarrow \frac{2\pi}{15} \text{ cubic units}$$

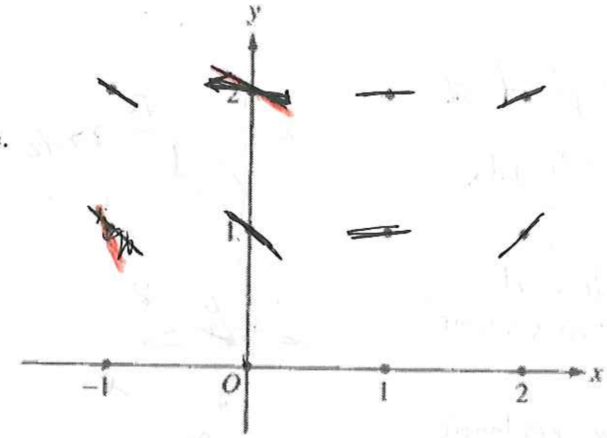
red slopes are the good ones, i dun goofed

D-DE4

Consider the differential equation $\frac{dy}{dx} = \frac{x-1}{y^2}$

3. On the axes provided, sketch a slope field at the points indicated.

4. While only some points are graphed, the slope field drawn in the previous problem is defined for many others. Describe all points in the xy-plane that have negative slope.



x	y	dy/dx
0	1	-1
0	2	-1/4
1	1	0
1	2	0
2	1	1
2	2	1/4
-1	1	-2
-1	2	-1/2

$\frac{dy}{dx} = \frac{x-1}{y^2}$

Always positive

So when is $x-1$ negative?

$x-1<0$

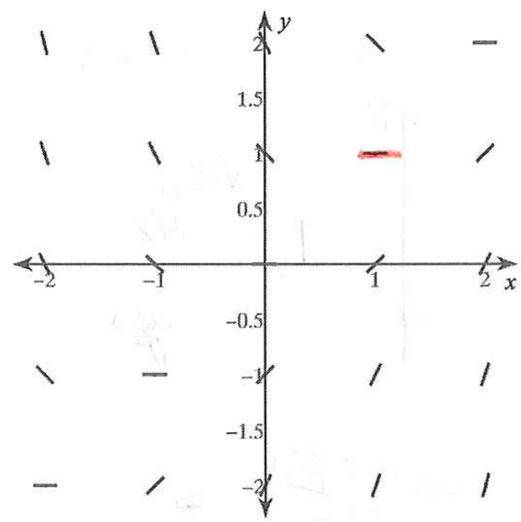
$\boxed{x<1}$

D-DE5

5. Choose the differential equation that could be represented by the given slope field.

A) $\frac{dy}{dx} = \frac{x}{y}$ B) $\frac{dy}{dx} = xy$

C) $\frac{dy}{dx} = x - y$ D) $\frac{dy}{dx} = x + y$



pick a representative pt like (1,1)

pt	A	B	C	D
(1,1)	1	1	0	2
	x	x	✓	x

{slope is 0}

Solutions N. M.

D-DE1 Practice Assessment

1. An illness is spreading through a population of N people. Let R represent the number of people with the illness. The rate with respect to time of people with the illness is growing is directly proportional to the product of the number of people with the illness and the square root of the population size. Write a differential equation that models this situation.

N

$$\frac{dR}{dt} = kR \cdot \sqrt{N}$$

D-DE3:

2. Consider the differential equation $y' = 2y - 3$. Find the general solution y.

$$y' = \cancel{d}\left(\frac{dy}{\cancel{dx}}\right) = (2y-3)\,dx$$

$$\frac{dy}{2y-3} = \frac{\cancel{(2y-3)}\,dx}{\cancel{2y-3}}$$

Sep. of Variables

$$\int \frac{1}{2y-3} \cdot dy = \int 1\,dx$$

Integrate

$$\frac{1}{2}\int 2 \cdot \frac{1}{2y-3}\,dy = x + C$$

$$\frac{1}{2}\ln|2y-3| + C = x + C$$

$$\frac{1}{2}\ln|2y-3| + \cancel{C} = x + C$$
$$-\cancel{C} \qquad -C$$

could be different.

$$\cancel{2}\left(\frac{1}{\cancel{2}}\ln|2y-3|\right) = (x + C)\,2$$

$$\ln|2y-3| = 2x + C$$

Exponentiate

$x^n \cdot x^m = x^{n+m}$

$$e^{2x+C} = 2y - 3$$

$$e^{2x} \cdot e^{C} = 2y - 3$$

$$\frac{Ce^{2x}}{2} + \frac{3}{2} = \frac{\cancel{2}y}{\cancel{2}}$$

$$Ce^{2x} + \frac{3}{2} = y$$

D-DE2: Consider the differential equation $\frac{dy}{dx} = 4y^2x$

3. Find the particular solution with initial condition (1,1/3)

$$\cancel{dx}\left(\frac{dy}{\cancel{dx}}\right) = (4y^2x)\,dx$$

$$\frac{dy}{y^2} = \frac{4\cancel{y^2}x\,dx}{\cancel{y^2}}$$

$$\int y^{-2}\,dy = \int 4x\,dx$$

$$\frac{y^{-1}}{-1} + C = 2x^2 + C$$

$$-\frac{1}{y} + \cancel{C} = 2x^2 + C \qquad -C$$

$$-\frac{1}{y} = 2x^2 + C$$

← General Solution

$$-\frac{1}{y} = 2x^2 + C$$

before solving for y, let's find C

plug in $\left(1, \frac{1}{3}\right)$

$$-\left(\frac{1}{\frac{1}{3}}\right) = 2(1)^2 + C$$

$$-3 = 2 + C$$

$$-2 \qquad -2$$

$$-5 = C$$

$$\cancel{-1}\left(-\frac{1}{y} = 2x^2 - 5\right)\cdot -1$$

$$\cancel{y}\left(\frac{1}{\cancel{y}}\right) = (5 - 2x^2)\,y$$

$$\frac{1}{5-2x^2} = \frac{\cancel{(5-2x^2)}\,y}{\cancel{5-2x^2}}$$

$$\boxed{\frac{1}{5-2x^2} = y}$$