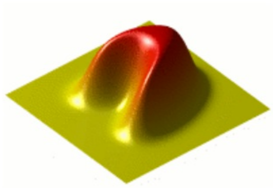
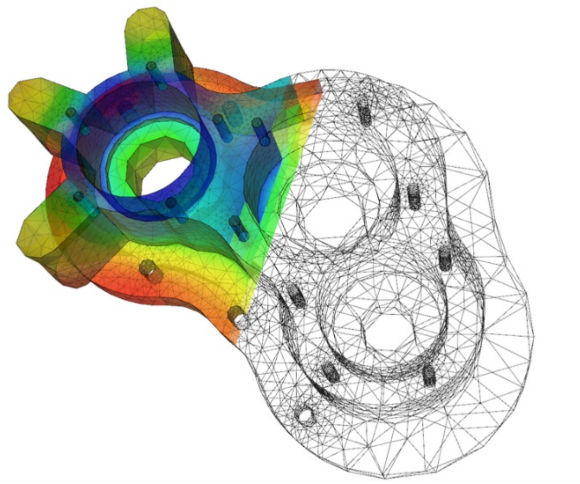
Differential Equations:

an equation relating a function with its derivative

Traditional Equation: x-->y
Differential Equation: x and or y --> y'

$$\frac{\partial u}{\partial t} - \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0$$





Most common technique for Solving Diff Eq in Calculus I:

Separation of Variables:

1.) "Dismantle" dy/dx by multiplying both sides by dx, or cross multiplying, etc.

2.) Group x terms on the same side of the equation as dx; y with dy

$\int_a^b$

3.) Once equation looks like: (math stuff) dy = (math stuff) dx integrate both sides ("antiderivative as operator")

4.) Don't forget +C !!!!!!!!!

5.) Group constants together; sometimes function can be left in implicit form

Example:

$y' = \dfrac{2x}{y}$

$$\left(\frac{dy}{\cancel{dx}} = \frac{2x}{y}\right) dx$$

$$\left(dy = \frac{2x}{\cancel{y}} \cdot dx\right) \cancel{y}$$

$$\int y\, dy = \int 2x\, dx$$

"integral as operator"

$$\frac{1}{2}y^2 + \cancel{C} = x^2 + C$$
$$\cancel{-C} \qquad\qquad -C$$

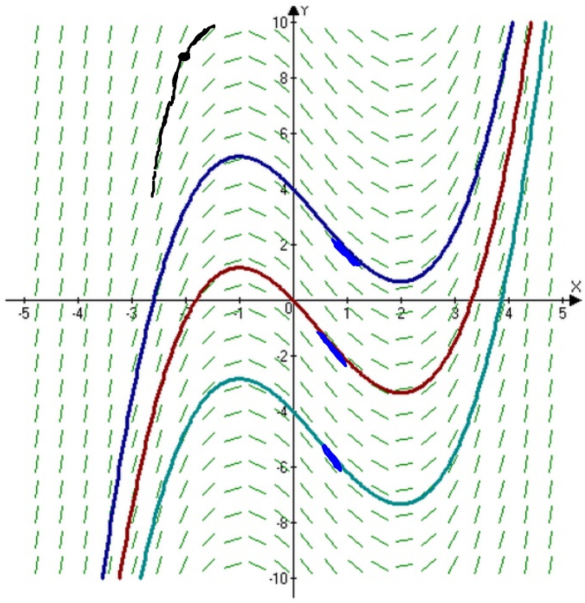
$$\left(\frac{1}{2}y^2 = x^2 + C\right)2$$

$$y^2 = 2x^2 + C$$

$$y = \pm\sqrt{2x^2 + C} \neq \sqrt{2x^2} + \sqrt{C}$$

General Solution to a Differential Equation:

Since you are taking an *indefinite* integral, the solution to a diff. eq will be a family of functions:



Find the general solution to the differential equation:

$(x^2+4)y' = xy$

$\left((x^2+4)\frac{dy}{\cancel{dx}} = xy\right)dx$

$\frac{\cancel{(x^2+4)}\,dy}{y\cancel{(x^2+4)}} = \frac{x\cancel{y}\,dx}{\cancel{y}(x^2+4)}$

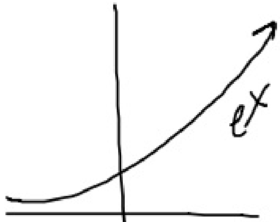
$\int \frac{1}{y}\,dy = \int \frac{x}{x^2+4}\cdot dx$

$\frac{1}{2}\int 2x\cdot\frac{1}{x^2+4}\,dx$

$\int =$ ''

$\ln|y| + C = \frac{1}{2}\ln(x^2+4) + C$

$\ln_e|y| = \frac{1}{2}\ln(x^2+4) + c$



$e^{\frac{1}{2}\ln(x^2+4)+c} = y$

$e^{\frac{1}{2}\ln(x^2+4)}\,e^{c} = y$

$\boxed{a^n\cdot a^m = a^{n+m}}$

a number

$C\cdot e^{\frac{1}{2}\ln(x^2+4)} = y$

$a\cdot\ln x = \ln x^a$

$C\,e^{\ln(x^2+4)^{1/2}} = y$

$C(x^2+4)^{1/2} = y$

Your turn:

$y' = \sin^2 y * \cos 2x$

$$\frac{dy}{dx} = \sin^2 y \cdot \cos 2x$$

$$\frac{dy}{\sin^2 y} = \frac{\cancel{\sin^2 y} \cdot \cos 2x \cdot dx}{\cancel{\sin^2 y}}$$

$$\frac{dy}{\sin^2 y} = \cos 2x \cdot dx$$

$$\frac{1}{\sin^2 y} dy = \cos 2x \, dx$$

$$\int \csc^2 y \, dy = \frac{1}{2}\int 2\cos 2x \, dx$$

$$-\cot y + C = \frac{1}{2} \sin 2x + C$$

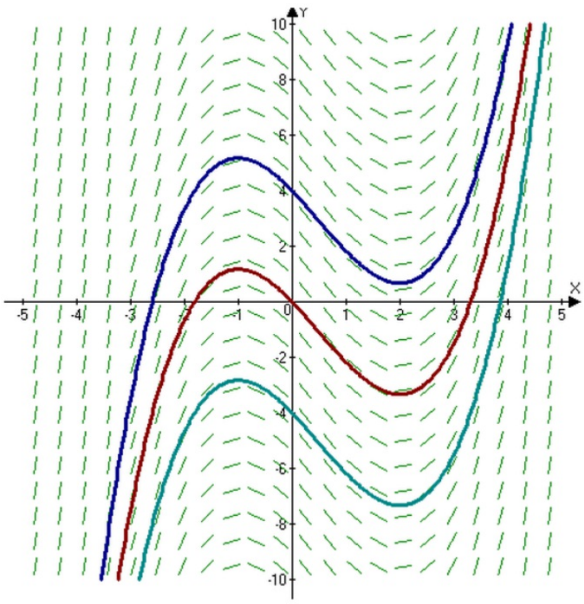
$$-\cot y = \frac{1}{2} \sin 2x + C$$

$$y = \cot^{-1}\left(C - \frac{1}{2}\sin 2x\right)$$

$\cot \theta = x$

$\theta = \cot^{-1}(x)$

Notice that constant of integration floating around in there?
With more information, we can find the **particular** solution. This information is typically a point, value, ordered pair, etc.



Find the particular solution