Limit Defi of the Derivative

The derivative, or instanteous rate of change, of a function $f(x)$ at $x=c$ is defined as

$$f'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

"f prime"

ex/ What is the (instant rate of change)
derivative of $f(x)$ @ $x=2$.

$$f(x) = 3x^2 - 2x + 5$$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$(a+b)^2$
$= a^2 + 2ab + b^2$

$$\lim_{h\to 0} \frac{3(2+h)^2 - 2(2+h) + 5 - 13}{h}$$

$$\lim_{h\to 0} \frac{3(4+4h+h^2) - 4 - 2h + 5 - 13}{h}$$

$$\lim_{h\to 0} \frac{\cancel{12} + 12h + 3h^2 \cancel{-4} - 2h \cancel{+5} \cancel{-13}}{h}$$

$$\lim_{h\to 0} \frac{12h + 3h^2 - 2h}{h}$$

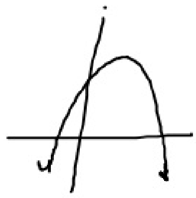
$$\lim_{h\to 0} \frac{\cancel{h}(3h+10)}{\cancel{h}}$$

$$\lim_{h\to 0} 3h + 10 = 10$$

Derivative as a function

$$\underbrace{f'(x)}_{\text{Derivative of } f(x)} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y = -5x^2 + 3$$



$$\underbrace{\frac{dy}{dx}}_{} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad *$$

means
"the derivative
of y
w.r.t. x"
same as
$f' = y'$

$$\lim_{h \to 0} \frac{-5(x+h)^2 + 3 - (-5x^2 + 3)}{h}$$

$$\lim_{h \to 0} \frac{-5(x^2 + 2xh + h^2) + 3 + 5x^2 - 3}{h}$$

$$\lim_{h \to 0} \frac{-\cancel{5x^2} - 10xh - 5h^2 \cancel{+3} + \cancel{5x^2} \cancel{-3}}{h}$$

$$\lim_{h \to 0} \frac{-10xh - 5h^2}{h}$$

$$\lim_{h \to 0} -10x - 5h = -10x$$

$$\underbrace{\frac{dy}{dx} = y' = f'(x)}_{\text{the derivative}} = -10x$$

$$y = 2x^2 + 2x$$

$$\frac{d}{dx} y = \frac{d}{dx}\left(2x^2 + 2x\right)$$

$$\frac{d}{dx} y = \frac{d}{dx}\left(2x^2\right) + \underbrace{\frac{d}{dx}\left(2x\right)}_{2}$$