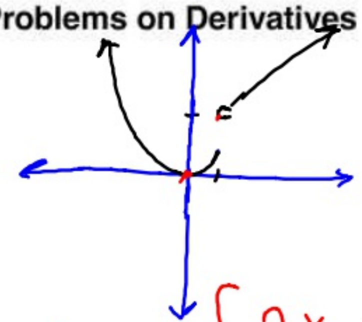
2.18 Sample A.P. Problems on Derivatives



650. Let $f(x) = \begin{cases} x^2 & x \le 1 \\ 2x & x > 1. \end{cases}$

$\lim_{x\to 1^+}$

a) Find $f'(x)$ for $x < 1$. $2x$

$\lim_{x\to 1^+} 2 = 2$

b) Find $f'(x)$ for $x > 1$. 2

$\lim_{x\to 1^-} 2x = 2$

c) Find $\lim_{x\to 1^-} f'(x)$. 2

d) Find $\lim_{x\to 1^+} f'(x)$. 2

$$f'(x) = \begin{cases} 2x, & x < 1 \\ 2, & x > 1 \end{cases}$$

e) Does $f'(1)$ exist? Explain. No. b/c f is not continuous @ x=1.

651. Let f be the function with derivative $f'(x) = \sin(x^2)$ and $f(0) = -1$.

a) Find the tangent line to f at $x = 0$.

b) Use your answer to part (a) to approximate the value of f at $x = 0.1$.

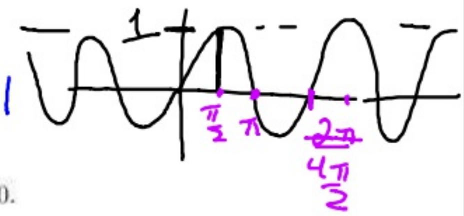
c) Is the actual value of f at $x = 0.1$ greater than or less than the approximation from part (b)? Justify your answer.

652 (1987AB). Let $f(x) = \sqrt{1 - \sin x}$. $f(0) = 1$

$(1-\sin x)^{1/2}$

a) What is the domain of f? $\mathbb{R}$.

$1 - \sin x \ge 0$, $-\sin x \ge -1$, $\sin x \le 1$, $x \in \mathbb{R}$.



b) Find $f'(x)$. $\frac{1}{2}(1-\sin x)^{-1/2} \cdot -\cos x$

$$\frac{-\cos x}{2\sqrt{1-\sin x}}$$

c) What is the domain of f'? $x \neq \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$

$1 - \sin x \neq 0$, $\sin x \neq 1$

d) Write an equation for the line tangent to the graph of f at $x = 0$. $m = -\frac{1}{2}$ $y - y_1 = m(x - x_1)$

653 (1994AB). Consider the curve defined by $x^2 + xy + y^2 = 27$.

a) Write an expression for the slope of the curve at any point (x, y).

b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

c) Find the points on the curve where the lines tangent to the curve are vertical.

654 (1994AB). A circle is inscribed in a square. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.

$\frac{dC}{dt} = 6$ $C = 2\pi \cdot r$

a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

$P = 4 \cdot s$

b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

$P' = 4\frac{ds}{dt}$