Goals: Implicit Differentiation

HW: p. 142

#1-15, 29-34

(due ~~Thurs.~~ FRI.)

Implicit Diff.

$f(x) = \underline{\underline{\quad}}$

Explicit $y = \pm\sqrt{25 - x^2}$

dep. Indy.

(explicit b/c dep. variable is isolated.)

Implicit

$$x^2 + y^2 = 25$$

find $\frac{dy}{dx}$.

$\boxed{\text{* the derivative as operator *}}$

"something you do to an eq."

$$\frac{d}{dx} x^2 + y^2 = \frac{d}{dx} 25$$

$$\frac{d}{dx} x^2 + \underbrace{\frac{d}{dx} y^2} = 0$$

$$2x + \underbrace{2y \cdot \frac{dy}{dx}}_{\frac{d}{dy} y^2} = 0$$

$$\frac{d}{dx} y^2$$

$$\frac{d}{dy} y^2 \cdot \frac{dy}{dx} \quad \text{CHAIN RULE.}$$

$$2y \cdot \frac{dy}{dx}$$

$$\cancel{2x} + 2y \frac{dy}{dx} = 0$$

$$-\cancel{2x} \qquad -2x$$

$$\frac{\cancel{2y}\frac{dy}{dx}}{\cancel{2y}} = \frac{-\cancel{2}x}{\cancel{2}y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$xy^2 - x^3y = 6$$

$$\frac{d}{dx}\, xy^2 - x^3y = \frac{d}{dx}\, 6$$

$$\frac{d}{dx}\underbrace{xy^2}_{f\,g} - \frac{d}{dx}\, \overset{f}{x^3}\,\overset{g}{y} = 0$$

$$\overset{f'}{1}\cdot \overset{g}{y^2} + \overset{f}{x}\cdot \overset{g'}{2y\frac{dy}{dx}}$$

$$-\left(3x^2\cdot y + x^3\cdot 1\frac{dy}{dx}\right) = 0$$

$$\cancel{y^2} + 2xy\frac{dy}{dx} - \cancel{3x^2y} - x^3\frac{dy}{dx} = 0$$

$$\underline{+y^2} \qquad \underline{+\cancel{3x^2y}}$$

$$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = -y^2 + 3x^2y$$

$$\frac{\frac{dy}{dx}\cancel{(2xy - x^3)}}{\cancel{2xy - x^3}} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

Resources

$$\frac{d}{dx}\, fg = f'g + fg'$$

$$\frac{d}{dx}\, y^2$$

ex $\quad 2y\cdot\frac{dy}{dx}$

$$\frac{d}{dx}\, \sin(y)$$

$$\cos(y)\frac{dy}{dx}$$

ex $\quad \frac{d}{dy}\, 3y^2$

$$6y$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

b.) $x = 1 \Rightarrow$ what is y?

$$xy^2 - x^3y = 6$$

$$1y^2 - 1^3y = 6$$

$$y^2 - y = \cancel{6}$$

$$\underline{\quad -6 \quad} \quad \underline{-6}$$

$$y^2 - y - 6 = 0$$

$$(y+2)(y-3) = 0$$

$$y = -2, \; y = 3$$

points.

$$\boxed{\begin{matrix} (1, -2) \\ (1, 3) \end{matrix}}$$

$\underline{(1,-2)}$

$$\frac{dy}{dx} = \frac{3(1^2)(-2) - (-2)^2}{2(1)(-2) - 1^3}$$

$$\frac{dy}{dx} = \frac{-10}{-5} = 2$$

$$\boxed{y + 2 = 2(x-1)}$$

$\underline{(1,3)}$

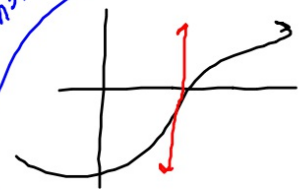
$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{0}{5} = 0$$

$$y - 3 = 0(x-1)$$

$$\boxed{y = 3}$$

c.) Vertical tangent line

set denom = 0



$$\frac{\text{rise}}{\text{run}} \quad \frac{\infty}{0} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$$2xy - x^3 = 0$$

$$x(2y - x^2) = 0$$

$$x = 0 \qquad 2y - x^2 = 0$$

$$2y = x^2$$

$$y = \frac{x^2}{2}$$