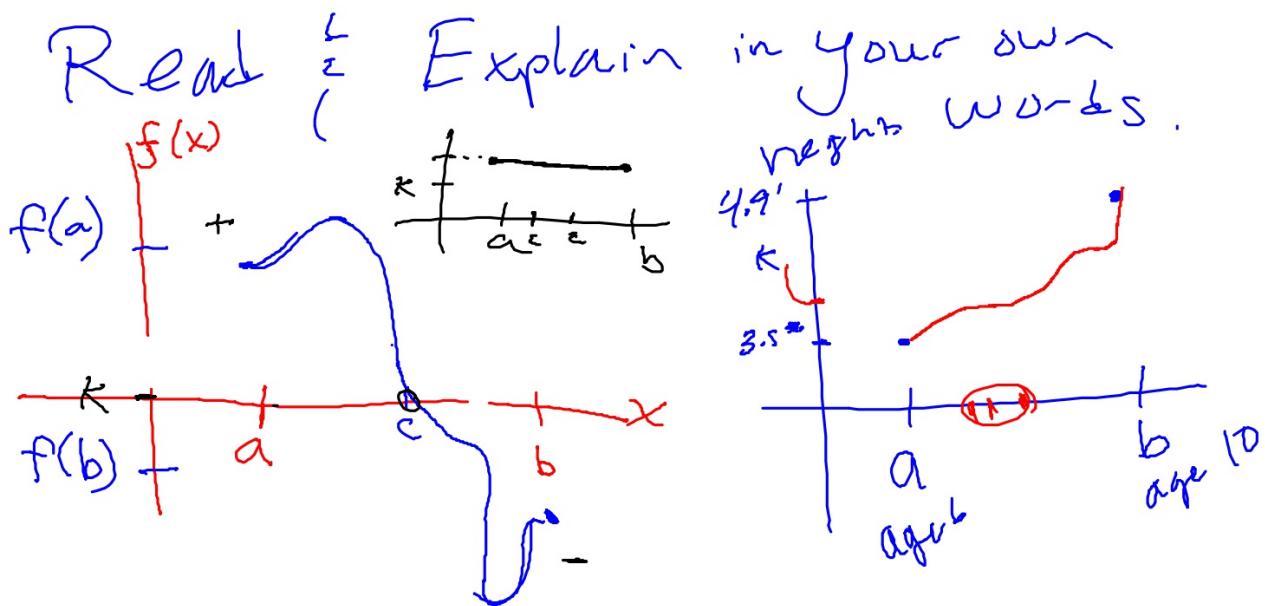


**THEOREM 1.13 Intermediate Value Theorem**

If  $f$  is continuous on the closed interval  $[a, b]$ ,  $f(a) \neq f(b)$  and  $k$  is any number between  $f(a)$  and  $f(b)$ , then there is at least one number  $c$  in  $[a, b]$  such that  $f(c) = k$ .

$$\begin{aligned} P &\rightarrow Q \\ \neg Q &\rightarrow \neg P \end{aligned}$$



$$f(s) = \frac{s+5}{s^2 - \frac{s}{2} - 3} \quad \lim_{t \rightarrow 0}$$

Let  $x=s$ .

$$\frac{x+s}{x^2 - \frac{x}{2} - 3}$$
$$\frac{x+s}{x^2 - 0.5x - 3} = \frac{x+5}{(x+1.5)(x-2)}$$

Inf. @  $x=-1.5$   $x=2$

$$\lim_{x \rightarrow -1.5} f = \frac{3.5}{0^+} = \pm\infty$$

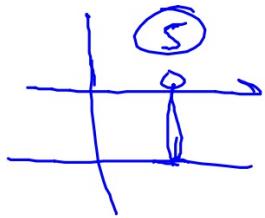
$\lim_{x \rightarrow 2^-}$  cont.

$\lim_{x \rightarrow 2} x^2 + 3x + 2 = f(2)$

$f(2) =$

$$\lim_{x \rightarrow 3} 5 = ?$$

(S)



$$\lim_{x \rightarrow 3} f(x) = 20$$

$$\lim_{x \rightarrow 3} g(x) = 30$$

1.)  $\lim_{x \rightarrow 3} 2f(x) =$

$$2 \cdot \lim_{x \rightarrow 3} f(x) = \boxed{40}$$

"Limit of a scalar multiple"

is the sc. mult. of the limit"

2.)  $\lim_{x \rightarrow 3} [f(x) + g(x)] = \lim_{x \rightarrow 3} f(x) + \lim_{x \rightarrow 3} g(x)$

"Limit of sum = sum of limits"

3.)  $\lim_{x \rightarrow 3} [f(x) \cdot g(x)] = \lim_{x \rightarrow 3} f(x) \cdot \lim_{x \rightarrow 3} g(x)$

"Limit of prod.  
is prod. of limits"

$$= 20 \times 30 = \boxed{600}$$

4.)  $\lim_{x \rightarrow 3} \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)}$ , if  $\lim_{x \rightarrow 3} g(x) \neq 0$

5.)  $\lim_{x \rightarrow 3} [f(x)]^n = \left[ \lim_{x \rightarrow 3} f(x) \right]^n$

$$\lim_{x \rightarrow 2} (x+5)^3 = \left[ \lim_{x \rightarrow 2} (x+5) \right]^{13}$$
$$= 7^{13}$$

$$\lim_{x \rightarrow 2} \sqrt[3]{(x-4)^2}$$

$$\lim_{\substack{x \rightarrow 2 \\ x \neq 2}} [(x-4)^2]^{1/3}$$

$$\lim_{x \rightarrow 2} (x-4)^{2/3} =$$

$$[(\lim_{\substack{x \rightarrow 2 \\ x \neq 2}} (x-4))]^{2/3} \rightarrow [(-2)^2]^{1/3} \rightarrow \sqrt[3]{4}$$

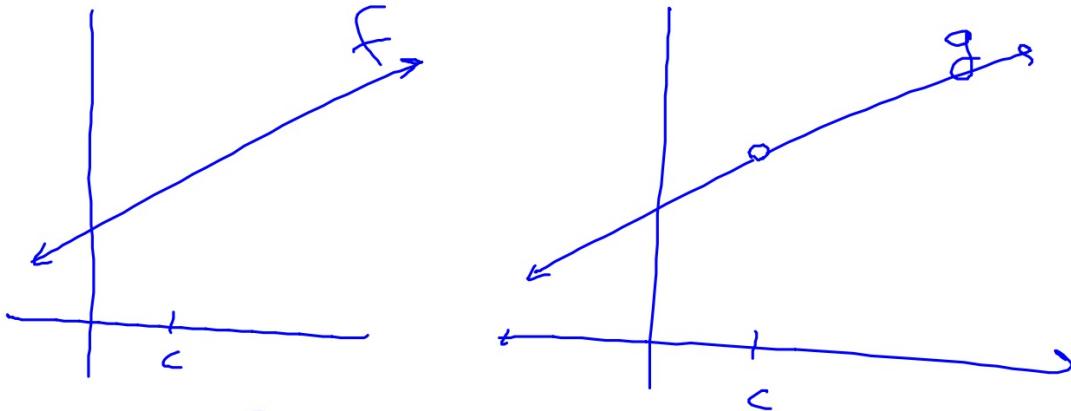
Limit of Composite

$$\lim_{x \rightarrow c} f(g(x)) = f\left(\lim_{x \rightarrow c} g(x)\right)$$

~~$\lim_{z \rightarrow 3} \cos(z^2 - 2z)$~~

$$= \cos\left(\lim_{z \rightarrow 3} z^2 - 2z\right)$$

$$= \cos(3) \approx -0.989$$



for all  $x \neq c$   
 $f(x) = g(x)$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} g(x)$$

P. 78: 75-8, 95, 100-103

(Sec 75  
for I.U.T.)  
the Wed D.S.





