

I-A1 or I-A2a

Practice Sols.

(NM)

$$1. \int \frac{3x^3 - x^4}{x^2} dx$$

$$\int \frac{3x^3}{x^2} - \frac{x^4}{x^2} dx$$

rewrite

$$\int 3x - x^2 dx$$

simplify

$$\frac{x^a}{x^b} = x^{a-b}$$

$$3 \frac{x^2}{2} - \frac{x^3}{3} + C$$

$$\text{integrate: } \int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\boxed{\frac{3}{2}x^2 - \frac{1}{3}x^3 + C}$$

$$2. \int -\cos(x) dx$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\therefore \int \cos(x) dx = \sin x$$

$$= -\sin(x) + C$$

$$3. \int x(2x+1)^2 dx$$

$$\int x(2x+1)(2x+1) dx \quad \cdot \text{expand}$$

$$\int x(4x^2 + 4x + 1) dx \quad \cdot \text{"foil" / distribute / etc}$$

$$\int 4x^3 + 4x^2 + x dx \quad \cdot \text{distribute}$$

$$\cancel{4 \cdot \frac{x^4}{4}} + 4 \cdot \frac{x^3}{3} + \frac{x^2}{2} + C$$

$$\boxed{x^4 + \frac{4}{3}x^3 + \frac{1}{2}x^2 + C}$$

$$4. \int \frac{1}{\sqrt[4]{x^7}} dx \quad \cdot \quad \sqrt[n]{x^a} \Rightarrow x^{a/n}$$

$$\int \frac{1}{x^{7/4}} dx$$

$$\int x^{-7/4} dx \quad \cdot \quad \frac{1}{x^n} \Rightarrow x^{-n}$$

$$\frac{x^{-3/4}}{-3/4} + C$$

• Rev. Power Rule

$$-\frac{4}{3} x^{-3/4} + C$$

$$\frac{-4}{3x^{3/4}} + C$$

$$5. \int 2e^x dx = \boxed{2e^x + C} \quad \text{!!}$$

Be sure you know all these rules:

$$\int x^n dx$$

$$\int \frac{1}{x} dx$$

$$\int \cos x dx$$

$$\int e^x dx$$

$$\int \sin x dx$$

$$\int \tan x dx$$

$$\int \sec^2 x dx$$

$$\int \cot x dx$$

$$\int \sec x \tan x dx$$

$$\int \sec x dx$$

$$\int \csc^2 x dx$$

$$\int \csc x dx$$

$$\int \csc x \cot x dx$$

6. $\int \frac{-25 \sec^2(5x)}{\tan(5x)} dx$

$\int \frac{1}{x} dx = \ln|x| + C$

hmm...

$\int -25 \sec^2(5x) \cdot \frac{1}{\tan(5x)} dx$

Want: $\sec^2(5x) : 5$
or $5 \sec^2(5x)$

$\frac{d}{dx} \tan(x) = \sec^2(x)$

$-5 \int \frac{-1}{5} \cdot -25 \sec^2(5x) \cdot \frac{1}{\tan(5x)} dx$

$-5 \int 5 \sec^2(5x) \cdot \frac{1}{\tan(5x)} dx$

$-5 \int \text{[scribble]} \cdot \frac{1}{\tan(5x)} dx$
can ignore this now

$\int \frac{1}{x} dx = \ln|x| + C$

$-5 \left[\ln|\tan(5x)| + C \right]$
integrate

$-5 \ln|\tan(5x)| + C$

$$7. \int \frac{36x^3}{(3x^4+4)^3} dx$$

$$\int 36x^3 \cdot \frac{1}{(3x^4+4)^3} dx$$

$$\int 36x^3 (3x^4+4)^{-3} dx$$

Want: $12x^3$

$$3 \int \frac{1}{3} 36x^3 (3x^4+4)^{-3} dx$$

$$3 \int 12x^3 (3x^4+4)^{-3} dx$$

$$3 \int \cancel{12x^3} (3x^4+4)^{-3} dx$$

Integrate

$$3 \left[\frac{(3x^4+4)^{-2}}{-2} + C \right]$$

$$-\frac{3}{2} (3x^4+4)^{-2} + C \Rightarrow$$

okay

$$\frac{-3}{2(3x^4+4)^2} + C$$

better ☺

$$\int x^{-3} dx = \frac{x^{-2}}{-2} + C$$

hmm...

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

8. $\int 5x \sqrt{4x-3} dx$

$\int 5x (4x-3)^{1/2} dx$

want: 4 ... 5x is not like 4 at all! :-

Let $u = 4x-3 \rightarrow u+3 = 4x \rightarrow \frac{1}{4}(u+3) = x$

$\frac{du}{dx} = 4 \rightarrow du = 4 \cdot dx \Rightarrow \frac{1}{4} du = dx$

$\int 5x (4x-3)^{1/2} dx$

$\int 5 \cdot \frac{1}{4}(u+3) (u)^{1/2} \cdot \frac{1}{4} du$

No more x's!

$5 \cdot \frac{1}{4} \cdot \frac{1}{4} \int (u+3) u^{1/2} du$

$\frac{5}{16} \int u^{3/2} + 3u^{1/2} du$

distribute

integrate rev. power rule

$x^a \cdot x^b = x^{a+b}$

$\frac{5}{16} \left[\frac{u^{5/2}}{5/2} + 3 \frac{u^{3/2}}{3/2} + C \right]$

$\frac{5}{16} \left[\frac{2}{5} u^{5/2} + 3 \cdot \frac{2}{3} u^{3/2} + C \right]$

$\frac{5}{16} \left[\frac{2}{5} u^{5/2} + 2u^{3/2} + C \right]$

$\frac{10}{80} u^{5/2} + \frac{10}{16} u^{3/2} + C$

$u = 4x-3$

$\frac{1}{8} (4x-3)^{5/2} + \frac{5}{8} (4x-3)^{3/2} + C$

C