4.18 Sample A.P. Problems on Integrals

1090. The figure shows the graph of the velocity of a model rocket for the first 12 seconds after launch.



- a) Assuming the rocket was launched from ground level, about how high did it go?
- b) Assuming the rocket was launched from ground level, about how high was the rocket 12 seconds after launch?
- c) What is the rocket's acceleration at t = 6 seconds? At t = 2 seconds?

1091. The graph of a function f consists of a semicircle and two line segments as shown below. Let $g(x) = \int_{1}^{x} f(t) dt$.

- a) Find g(1).
- b) Find g(3).
- c) Find g(-1).
- d) Find all the values of x on the open interval (-3, 4) at which g has a relative maximum.



- e) Write an equation for the line tangent to the graph of g at x = -1.
- f) Find the x-coordinate of each point of inflection of the graph of g on the open interval (-3, 4).
- g) Find the range of g.

1092. An automobile accelerates from rest at $1 + 3\sqrt{t}$ miles per hour per second for 9 seconds.

- a) What is its velocity after 9 seconds?
- b) How far does it travel in those 9 seconds?

1093. Find the function f with derivative $f'(x) = \sin x + \cos x$ whose graph passes through the point $(\pi, 3)$.

We have knowledge of the past but cannot control it; we may control the future but not have knowledge of it. -Claude Shannon

1094 (1989BC). Let f be a function such that f''(x) = 6x + 8.

- a) Find f(x) if the graph of f is tangent to the line 3x y = 2 at the point (0, -2).
- b) Find the average value of f(x) on the closed interval [-1, 1].

1095 (1999AB, Calculator). A particle moves along the y-axis with velocity given by $v(t) = t \sin(t^2)$ for $t \ge 0$.

- a) In which direction (up or down) is the particle moving at time t = 1.5? Why?
- b) Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing at t = 1.5?
- c) Given that y(t) is the position of the particle at time t and that y(0) = 3, find y(2).
- d) Find the total distance traveled by the particle from t = 0 and t = 2.

1096 (1990BC). Let f and g be continuous functions with the following properties:

i)
$$g(x) = A - f(x)$$
 where A is a constant
ii) $\int_{1}^{2} f(x) dx = \int_{2}^{3} g(x) dx$
iii) $\int_{2}^{3} f(x) dx = -3A$

- a) Find $\int_{1}^{3} f(x) dx$ in terms of A.
- b) Find the average value of g(x) in terms of A over the interval [1,3].
- c) Find the value of k if $\int_0^1 f(x+1) \, dx = kA$.

1097 (1994AB, Calculator). Let $F(x) = \int_0^x \sin(t^2) dt$ for $0 \le x \le 3$.

- a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0, 1] to approximate F(1).
- b) On what interval is F increasing?
- c) If the average rate of change of F on the closed interval [1,3] is k, find $\int_1^3 \sin(t^2) dt$ in terms of k.

1098 (1991BC). A particle moves on the x-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = 12t^2 - 36t + 15$.

- a) Find the position x(t) of the particle at any time $t \ge 0$.
- b) Find all values of t for which the particle is at rest.
- c) Find the maximum velocity of the particle for $0 \le t \le 2$.
- d) Find the total distance traveled by the particle from t = 0 to t = 2.