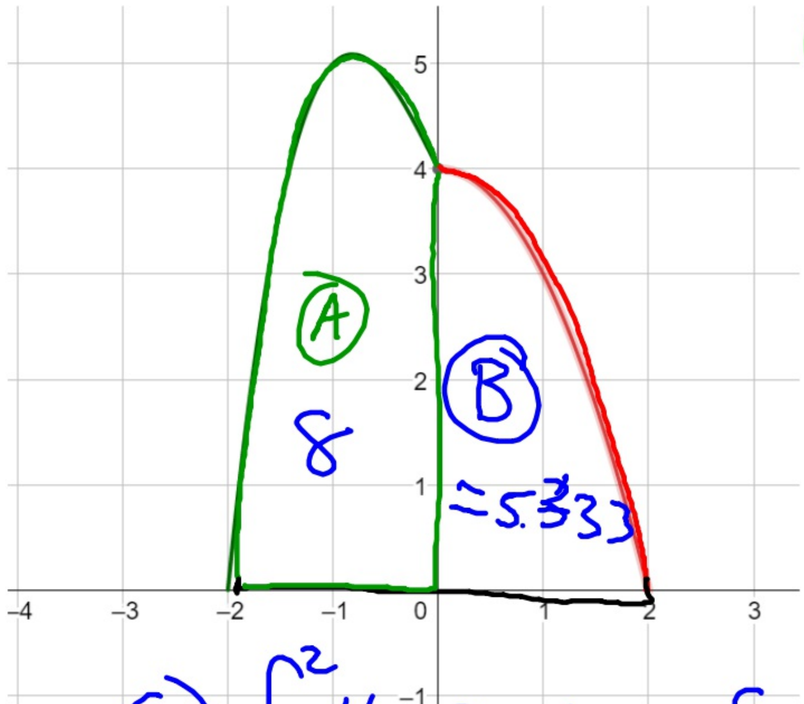
Good afternoon

Warm up: find the area of the region

$$f(x) = \begin{cases} x^3 - 2x + 4 & x \le 0 \\ 4 - x^2 & x > 0 \end{cases}$$



A : $\int_{-2}^{0}(x^3 - 2x + 4)\,dx$

$\left[\frac{1}{4}x^4 - x^2 + 4x\right]_{-2}^{0}$

$[0] - [4 - 4 + \overset{-8}{\cancel{4(-2)}}]$

$0 - [-8]$

8

B $\int_0^2 4 - x^2\,dx = \left[4x - \frac{1}{3}x^3\right]_0^2 \Rightarrow \left(8 - \frac{8}{3}\right) - 0$

$\frac{16}{3}$

A+B = $13.\overline{3}$ $\frac{40}{3}$

Let $g(x)=\int_x^{x^2}\sqrt{t}\,dt$

Find g'(x), intervals of increase, and inflection point(s), if any.

$$g(x)=\int_x^0\sqrt{t}\,dt+\int_0^{x^2}\sqrt{t}\,dt$$

$$\int_a^b=\int_a^c+\int_c^b$$

$$g(x)=-\int_0^x\sqrt{t}\,dt+\int_0^{x^2}\sqrt{t}\,dt$$

$$g'(x)=-\sqrt{x}+\sqrt{x^2}\cdot 2x$$

$$\boxed{g'(x)=2x^2-x^{\frac{1}{2}}}$$

$$2x^2-x^{1/2}=0$$

$$x^{\frac{1}{2}}\left(2x^{3/2}-1\right)=0$$

$x=0$

$$2x^{3/2}-1=0$$

$$2x^{3/2}=1$$

$$\left(x^{3/2}\right)^2=\left(\tfrac{1}{2}\right)^2$$

$$x^3=\frac{1}{4}\Rightarrow x=\sqrt[3]{1/4}\approx 0.630$$

g': ? | − | +

x: 0 .630

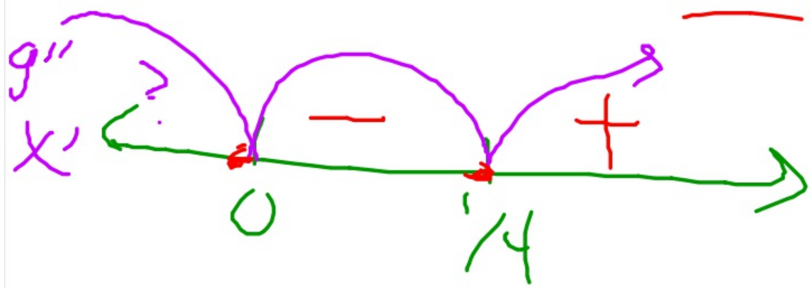
$\boxed{\text{g increases over } (.630, \infty) \text{ b/c } g'>0}$

$$g'(x) = 2x^2 - x^{\frac{1}{2}}$$

$$g''(x) = 4x^1 - \frac{1}{2}x^{-1/2} = 0$$

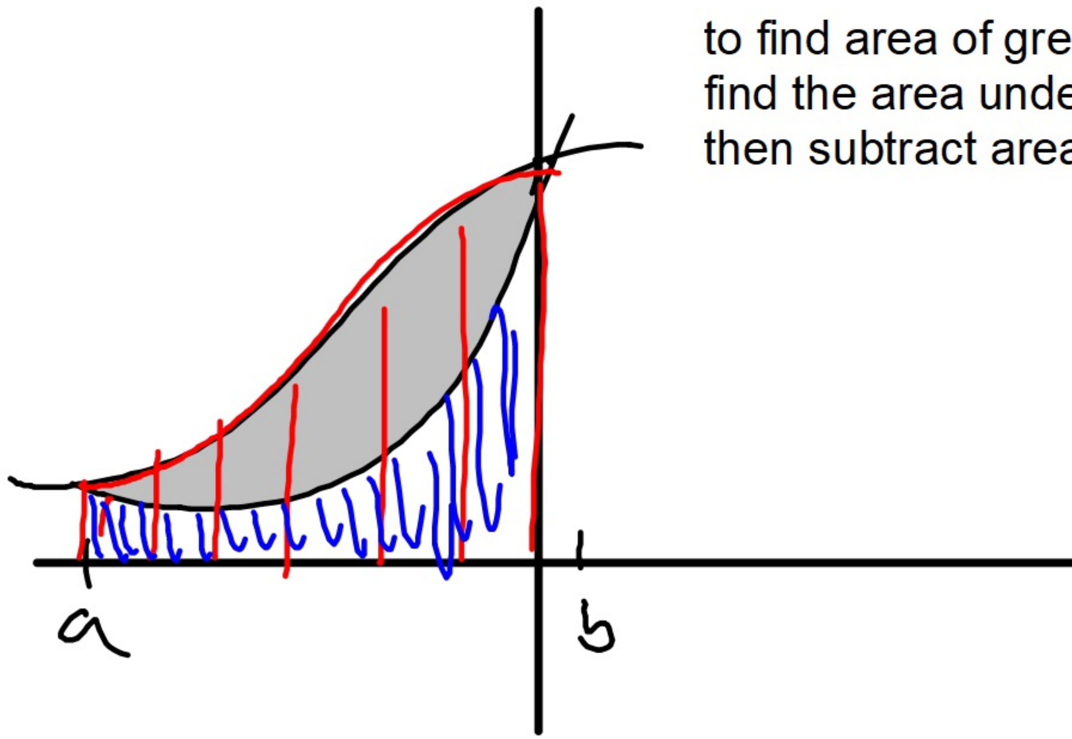
$$x^{-1/2}\left(4x^{3/2} - \frac{1}{2}\right) = 0$$

$x = 0$ $\qquad$ $x = \frac{1}{4}$ terrace points.

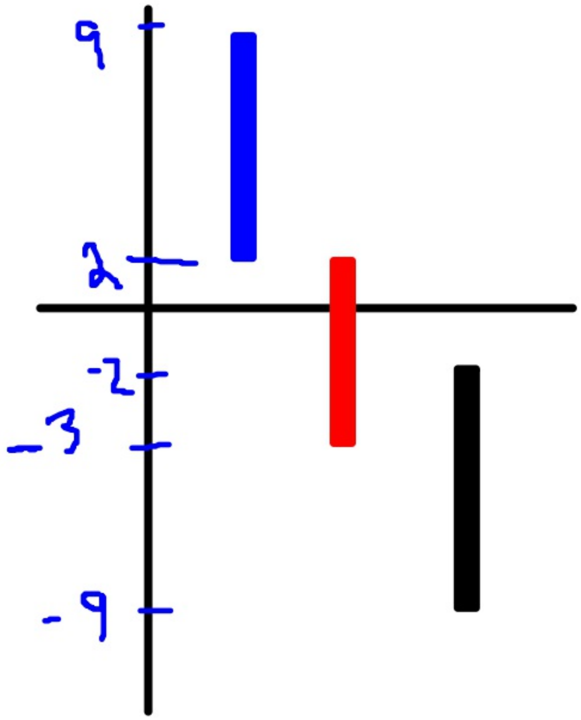


g has an I.P. @ $x = 1/4$
b/c g'' changes sign.

A preview of Area Between Curves



to find area of grey
find the area under top curve (red)
then subtract area under bottom curve (blue)



How 'tall' are these rectangles?

Blue: 9 -2 = 7 units tall
Red: 2 - -3 = 5 units tall
Black: -2 - -9 = 7 units tall

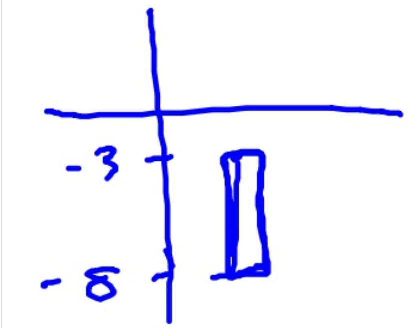
the rectangle's height is always top minus bottom

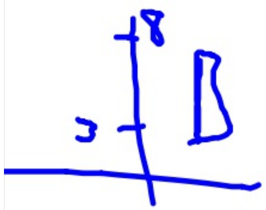
A preview of Area Between Curves

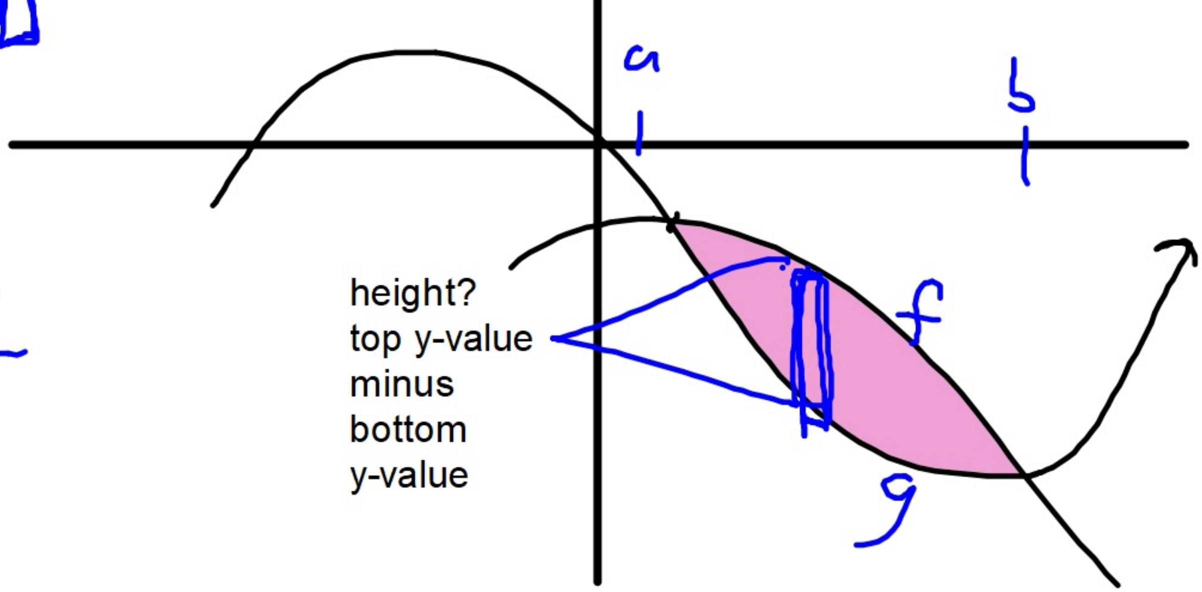
rectangle height

rectangle base

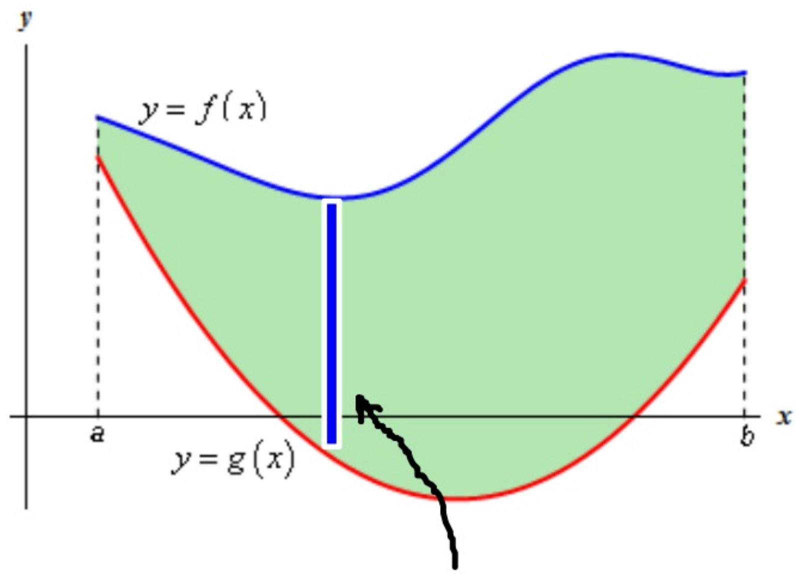
$$\int_a^b [f(x) - g(x)]\,dx$$







Even when some regions are negative:



height of this rectangle: f(x) - g(x)

base of this rectangle: Δx

area of one rectangle: f(x)-g(x) Δ x

therefore area of entire region is sum of infinite rectangles

$$\int_a^b f(x)-g(x)\,dx$$

HW: prepare for assessment
watch and take notes on video at mcalc.weebly.com

Want extra practice?
I-A4a: p288 #35-44
I-U4: p.290 #81-92
I-U7: p 274 #41-44
I-U5: p. 313 #43-48
I-U9: p. 290 #73-74