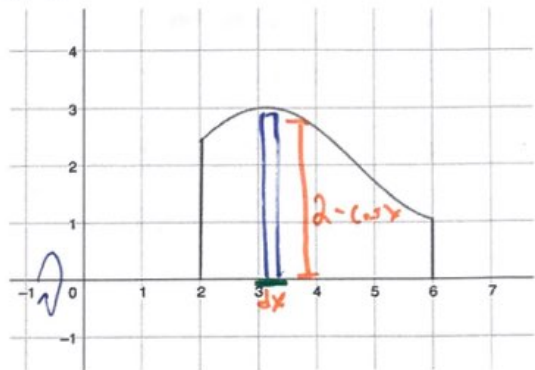1. Find the volume of the solid generated by revolving the region bounded by $f(x) = 2 - \cos x$ and the vertical lines $x = 2$ and $x = 6$ about the x-axis. ~~Show all work.~~
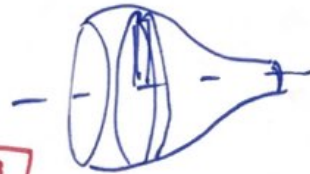
- Vol. of 1 cylinder: $\pi r^2 \cdot h$
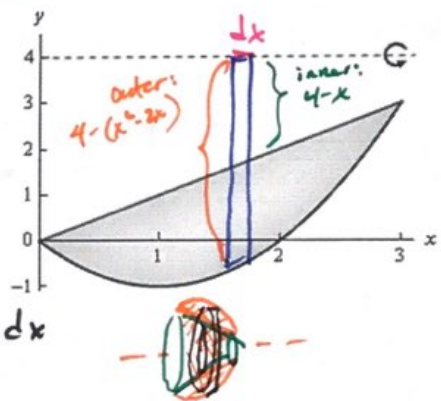
$$V = \pi (2 - \cos x)^2 \cdot dx$$

- Vol. of all cylinders

$$V = \int_2^6 \pi (2 - \cos x)^2 \, dx \implies \pi \int_2^6 (2 - \cos x)^2 \, dx$$

Calc. $\rightarrow$ $\boxed{22.810\pi \approx 71.659 \, u^3}$

2. Set-up a single integral to calculate the volume of the solid generated when the region bounded by $f(x) = x^2 - 2x$ and $g(x) = x$ is revolved around the axis $y = 4$. Then use a calculator to find that volume.
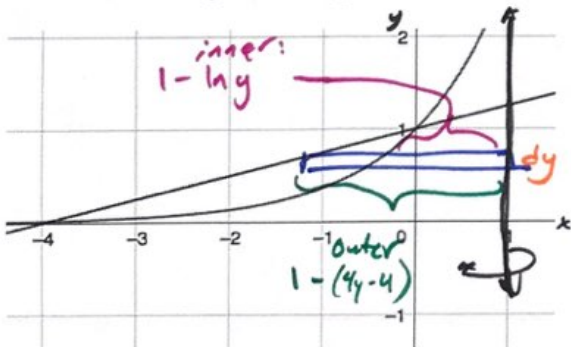
$$V = \pi \int_0^3 (4 - (x^2 - 2x))^2 - (4 - x)^2 \, dx$$

inner rad. I

$$V = \pi \cdot \int_0^3 (4 - x^2 + 2x)^2 - (4 - x)^2 \, dx$$

$\boxed{30.6\pi \approx 96.133 \, u^3}$
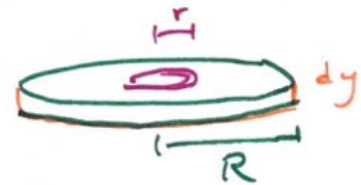
$\dfrac{153\pi}{5}$

outer rad.

$(\pi R^2 - \pi r^2) \, dx$

3. Let R be the region bounded by $g(x) = e^x$ and $h(x) = \frac{1}{4}x + 1$. Find the volume of the solid formed by revolving R about the vertical line $x = 1$. (Calc ok)

inner: $1 - \ln y$

outer: $1 - (4y - 4)$

$y = e^x$ $y = \frac{1}{4}x + 1$

$\ln y = \ln e^x$ $y - 1 = \frac{1}{4}x$

$\ln y = x$ $4y - 4 = x$

$$V = \pi \int_0^1 (1 - (4y - 4))^2 - (1 - \ln y)^2 \, dy$$

$$V = \pi \int_0^1 (1 - 4y + 4)^2 - (1 - \ln y)^2 \, dy$$

$$V = \pi \int_0^1 (5 - 4y)^2 - (1 - \ln y)^2 \, dy$$

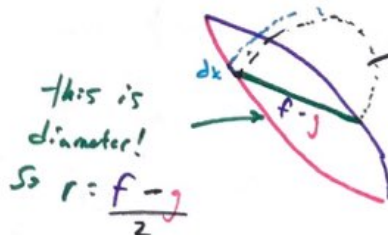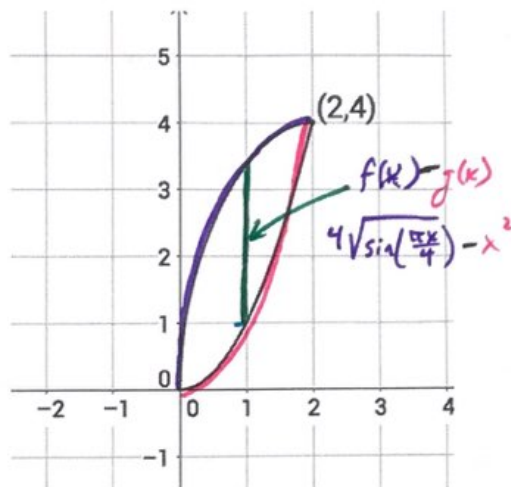$\boxed{5.333\pi \approx 16.755 \, u^3}$

I-A5c
Let R be the first-quadrant region enclosed by

$f(x) = 4\sqrt{\sin(\frac{\pi x}{4})}$ and $g(x) = x^2$.

4. Let R be the base of a solid whose cross-sections perpendicular to the x-axis are semicircles. Find the volume of this solid. (Calc ok)

(2,4)

$f(x) - g(x)$

$4\sqrt{\sin(\frac{\pi x}{4})} - x^2$

this is diameter!

So $r = \frac{f-g}{2}$

dx $f - g$

Area? $A = \frac{1}{2}\pi r^2$
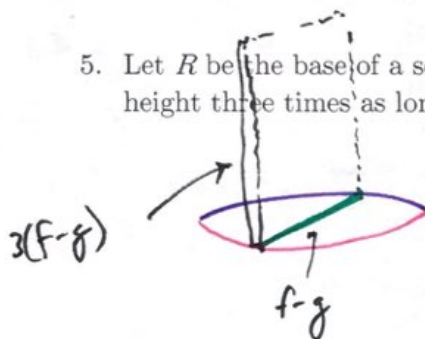
$A = \frac{\pi}{2}\left(\frac{f-g}{2}\right)^2$

$A = \frac{\pi}{2} \cdot \frac{(f-g)^2}{4}$

$A = \frac{\pi}{8}(f-g)^2$

$V = \int_0^2 \frac{\pi}{8}(f-g)^2 \, dx \Rightarrow \frac{\pi}{8}\int_0^2 \left(4\sqrt{\sin(\frac{\pi x}{4})} - x^2\right)^2 dx$

$\boxed{2.676 \, u^3}$

5. Let R be the base of a solid whose cross-sections perpendicular to the x-axis are rectangles with height three times as long as the base. Find the volume of this solid. (Calc ok)

$3(f-g)$

$f-g$

Area? $A = (f-g) \cdot 3(f-g)$
              base   height

$A = 3(f-g)^2$

$V = \int_0^2 3(f-g)^2 \, dx = 3\int_0^2 \left(4\sqrt{\sin(\frac{\pi x}{4})} - x^2\right)^2 dx$

$\boxed{20.441 \, u^3}$