Good afternoon: warm up in notes

$$\int \frac{1}{4x+8}\,dx$$

$$\frac{1}{4}\int 4\,\frac{1}{4x+8}\,dx$$

want: 4

$$\frac{1}{4}\left[\ln|4x+8|+C\right]$$

$$\boxed{\frac{1}{4}\ln|4x+8|+C}$$

$$\ln\sqrt[4]{|4x+8|}+C$$

$$\int \tan(3x)\,dx$$

$$\int \frac{\sin(3x)}{\cos(3x)}\,dx$$

$$-\frac{1}{3}\int -3\sin(3x)\,\frac{1}{\cos(3x)}\,dx$$

want: $-3\sin(3x)$

$$-\frac{1}{3}\left[\ln|\cos(3x)|+C\right]$$

$$-\frac{1}{3}\ln|\cos(3x)|+C$$

$$\frac{1}{3}\ln\left|[\cos(3x)]^{-1}\right|+C$$

$$\boxed{\frac{1}{3}\ln|\sec(3x)|+C}$$

$$\ln\sqrt[3]{|\sec(3x)|}+C$$

$$\log b^a \rightleftarrows a\log b$$

reminders:

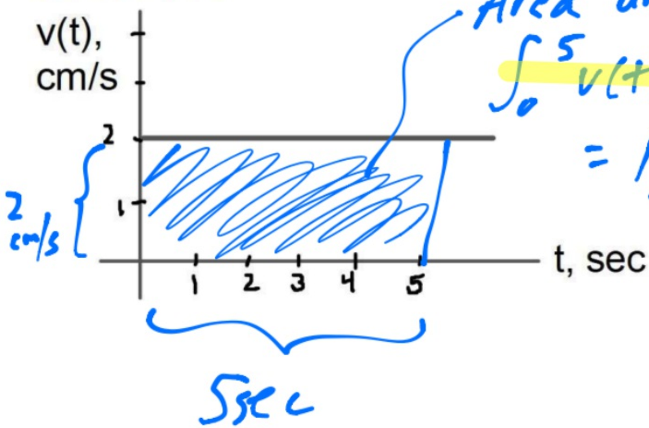
- assess: Friday -tutoring/retakes tomorrow 4-5p

Looking back at where we ended Friday:

Integration as Accumulation
A snail travels 2 cm/s along a stick. After 5 seconds, how far has it traveled?

Area under v(t)

$$\int_0^5 v(t)\,dt = \underline{10 \text{ cm}}$$



If its starting position was 6 cm from a ladybug, what is its position at t=8?

$t=0$ 16c $t=8$

6cm

$x(8) = 22$ cm

$x(0) = 6$ cm

↑ position

$x(8) = 6 + \int_0^8 v(t)\,dt = 22$ cm

Note: this 16 cm. is $\int_0^8 v(t)\,dt$

Also = to $x(8) - x(0)$

$22 - 6 = 16$

If an object moving in a straight line has velocity $v(t)=\sqrt{t}$ m/s, how far does it travel in the first 4 seconds?

position @ $t=8$
minus
position @ $t=0$

Answer in meters.

$$\begin{tikzcd} P \arrow[d, bend right, "\frac{d}{dt}"'] \\ v \arrow[u, bend right, "\int"'] \arrow[d, bend right, "\frac{d}{dt}"'] \\ a \arrow[u, bend right, "\int"'] \end{tikzcd}$$

$$v(t) = t^{1/2}$$

$$\frac{\text{change pos.}}{\text{change time}} \rightarrow \frac{dx}{dt} = t^{1/2}$$

$$dx = t^{1/2}\,dt$$

$$\int dx = \int t^{1/2}\,dt$$

$$x = \frac{2}{3}t^{3/2} + c \quad \text{general solution}$$

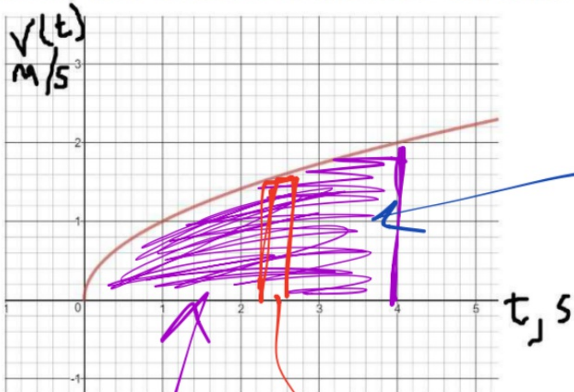
$$x(4) = \frac{2}{3}(4)^{3/2} + c = 5\tfrac{1}{3} + c$$

$$x(0) = 0 + c = 0 + c$$

$$x(4) - x(0) = 5\tfrac{1}{3} \cancel{+c-c} = 5\tfrac{1}{3} \text{ meter}$$

(new??)

If an object moving in a straight line has velocity $v(t)=\sqrt{t}$ m/s, how far does it travel in the first 4 seconds?



$x(4) - x(0)$

$= 5\,\frac{1}{3}$

$$\int_0^4 \sqrt{t}\, dt$$

Area = "meters"

The Fundamental Theorem of Calculus, Part II

Suppose F(x) is an antiderivative of f(x).
Then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

*not yet proven! we will need part 1 for that

Approximate $\int_4^9 \sqrt{x}\,dx$ using 5 trapezoids of equal width.

Estimate from Weds: 12.660

$$\int_4^9 x^{1/2}\,dx = \left[\frac{2}{3}x^{3/2}\right]_4^9$$

$$\frac{2}{3}\left(\sqrt{9}\right)^3 - \left[\frac{2}{3}\left(\sqrt{4}\right)^3\right]$$

$$18 - \frac{16}{3} = \frac{38}{3} = \boxed{12\frac{2}{3} \quad 12.\overline{6}}$$

Suppose F(x) is an antiderivative of f(x).
Then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

How to evaluate a definite integral:

1 Find the antiderivative of the integrand (leave off the +C)
2 Plug in b and plug in a
3 Subtract F(b)-F(a)
4 yay thats it

An example:

$$\int_{e^2}^{e^5} \frac{2}{x}\,dx = 2\int_{e^2}^{e^5} \frac{1}{x}\,dx$$

$$2\left[\ln|x|\right]_{e^2}^{e^5}$$

$$2\left[\ln e^5 - \ln e^2\right]$$

$$2\left[5 - 2\right]$$

$$2(3) = 6$$

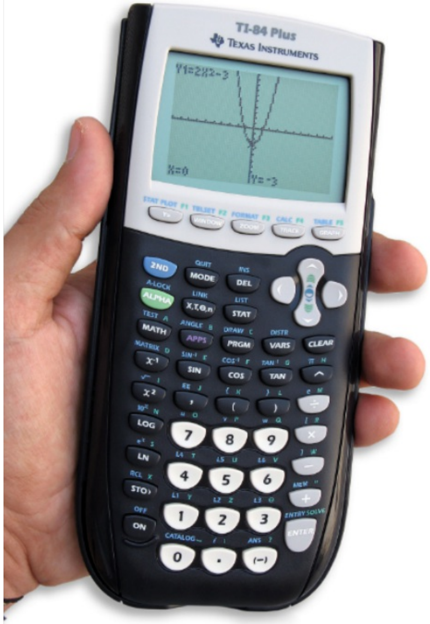
Another:

$$\int_{-3}^{2} \frac{1}{5}x^3 - \frac{1}{10}x^2\,dx$$

$$\left[\frac{1}{20}x^4 - \frac{1}{30}x^3\right]_{-3}^{2}$$

$$\left[\frac{1}{20}(2)^4 - \frac{1}{30}(2)^3\right] - \left[\frac{1}{20}(-3)^4 - \frac{1}{30}(-3)^3\right]$$

$$\frac{8}{15} - \frac{99}{20} = -\frac{53}{12}$$



Checking answer

$\boxed{\text{MATH}}$

9

HW

p. 288 #5-20