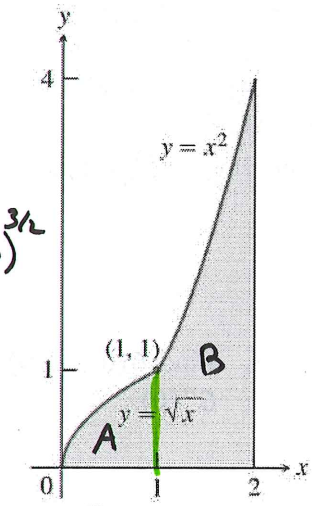
I-A4a

1. Find the exact area of the shaded region. Show all work.



(Recall that a Definite Integral is defined to Calculate area under a curve.)

Region A $\int_0^1 \sqrt{x}\,dx = \int_0^1 x^{1/2}dx = \left[\frac{2}{3}x^{3/2}\right]_0^1 = \frac{2}{3}(1)^{3/2} - \frac{2}{3}(0)^{3/2}$

$= \frac{2}{3} - 0$

$= 2/3$

Region B: $\int_1^2 x^2\,dx = \left[\frac{1}{3}x^3\right]_1^2 = \frac{1}{3}(2)^3 - \frac{1}{3}(1)^3$

$= \frac{8}{3} - \frac{1}{3}$

$= 7/3$

$\frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$ whoa!

I-U4

Let $f(x) = \int_3^{2x} 2t^2 - 3t - 2\,dt$.

2. Find $f'(x)$. Simplify your answer.

$f'(x) = [2(2x)^2 - 3(2x) - 2]\cdot 2 = [8x^2 - 6x - 2]\cdot 2 \Rightarrow 16x^2 - 12x - 4 = f'(x)$

3. Find all intervals where $f(x)$ is increasing. Justify your answer. See pg 2 for # 3

Find c.n.

$f'(x) > 0$

$0 = f'(x) = 16x^2 - 12x - 4 = 4(4x^2 - 3x - 1) \Rightarrow 4(4x+1)(x-1) = 0$

I-U7

Suppose $f(x)$ and $h(x)$ are continuous functions such that $\int_1^9 f(x)\,dx = -1$, $\int_7^9 f(x)\,dx = 5$, $\int_7^9 h(x)\,dx = 4$.

4. $\int_9^7 [h(x) - f(x)]\,dx \Rightarrow -\int_7^9 h(x) - f(x)\,dx = -\left[\int_7^9 h(x)dx - \int_7^9 f(x)dx\right]$

$-[4 - 5] = -[-1] = 1$

5. $\int_1^7 f(x)\,dx = \int_1^9 f(x)\,dx + \int_9^7 f(x)\,dx$

$= \int_1^9 f(x)\,dx - \int_7^9 f(x)\,dx$

I-U5

6. $\int_4^9 2x - \frac{1}{\sqrt{x}}\,dx$ $= -1 - 5 = -6$

7. If $\int_{-2}^2 (x^3 + k)\,dx = 16$, then what is the value of k?

See page 2 for #6-7

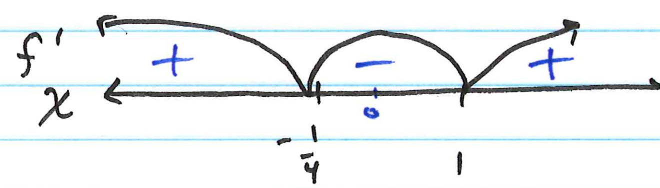
3. From #2: $f'(x) = 16x^2 - 12x - 4$

$f(x)$ increasing? Need $f'(x) > 0$

Find C.N. set $f'(x) = 0$

$$16x^2 - 12x - 4 = 0$$
$$4(4x^2 - 3x - 1) = 0$$
$$4(4x+1)(x-1) = 0$$

$x = -1/4$ $\quad$ $x = 1$ $\quad$ C.N.



$\boxed{\text{f increasing over } (-\infty, -1/4) \text{ and } (1, \infty) \text{ because } f'(x) > 0.}$

6. $\int_4^9 2x - \frac{1}{\sqrt{x}}\, dx$

$\int_4^9 2x - x^{-1/2}\, dx$

$\left[x^2 - 2x^{1/2}\right]_4^9$

$\left[x^2 - 2\sqrt{x}\right]_4^9$

$(9^2 - 2\sqrt{9}) - (4^2 - 2\sqrt{4})$

$(81 - 2\cdot 3) - (16 - 2\cdot 2)$

$75 - 12$

63

$\boxed{7.}$ $\int_{-2}^{2} x^3 + k\, dx = 16$ $\qquad$ $k = ?$

$\left[\frac{1}{4}x^4 + kx\right]_{-2}^{2} = 16$

$\left(\frac{1}{4}(2)^4 + 2\cdot k\right) - \left(\frac{1}{4}(-2)^4 - 2k\right) = 16$

$\left(\frac{1}{4}\cdot 16 + 2k\right) - \left(\frac{1}{4}\cdot 16 - 2k\right) = 16$

$\cancel{4} + 2k - \cancel{4} + 2k = 16$

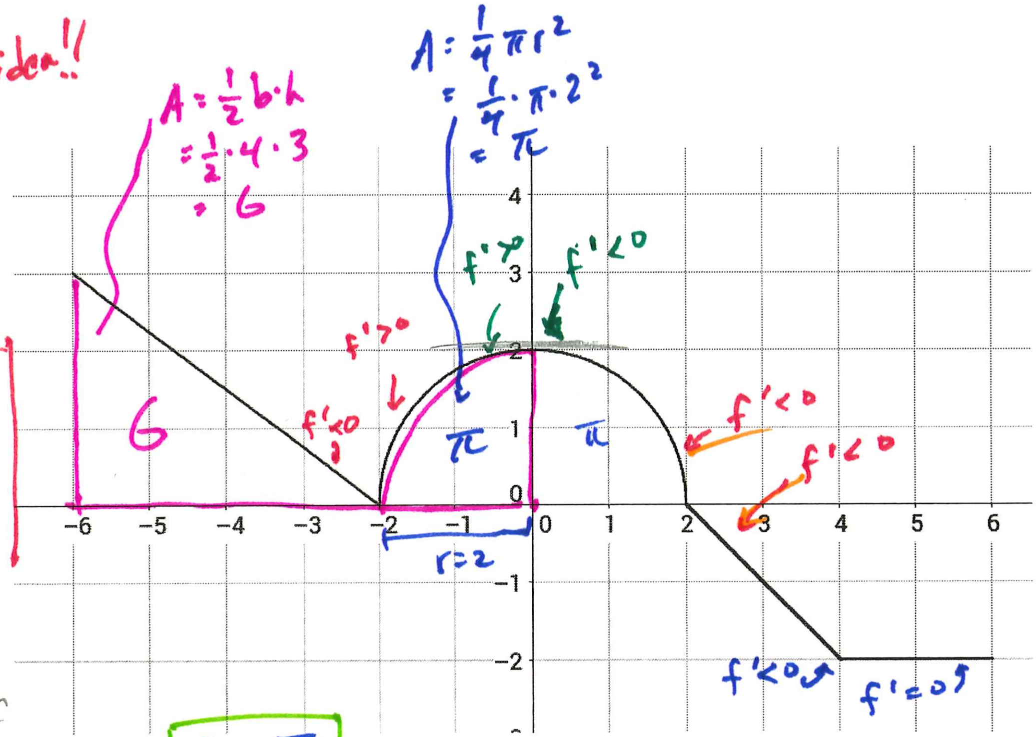
$4k = 16$

$\boxed{k = 4}$

Key idea!!

I-U9
The function $f(t)$ is shown over $[-6,6]$ and consists of line segments and a semicircle.
Let $G(x) = \int_{-6}^{x} f(t)\,dt$

FTC
$G'(x) = f(x)$ ← value of f
$G''(x) = f'(x)$ ← slope of f.



8. Find $G(0)$, $G'(0)$, and $G''(0)$.

$G(0) = \int_{-6}^{0} f(t)\,dt$ → Area under f from −6 to 0. → $\boxed{6 + \pi}$

$G'(0) = f(0)$ → Value of f when $x = 0$ = $\boxed{2}$

$G''(0) = f'(0)$ → Slope of f when $x = 0$ = $\boxed{0}$ (Horizontal tangent)

9. Find the relative maxima of $G(x)$, if any, over [-6,6]. Justify your answer.

Need $G'(x) = 0$ or undefined, sign change in G' from + → −.

Well, $G'(x) = f(x)$, so where is $f(x) = 0$? At $x = -2$ and 2. Critical Numbers

@ $x = -2$ Sign change? No. + → +

@ $x = 2$ Sign change? yes, + → − ✓

G has a rel. max @ $x = 2$. The value of $G(2)$ here is $\boxed{6 + 2\pi}$

10. Find any points of inflection of $G(x)$. Justify your answer.

Need $G''(x) = 0$ or undefined and sign change.

Well, $G''(x) = f'(x)$, so where does f change from inc. to dec. or vice versa?

@ $x = -2$ $f'(-2)$ undefined. Change: − → + ✓

@ $x = 0$ $f'(0) = 0$; Change: + → − ✓

@ $x = 2$ $f'(2)$ undefined; change: − → − ✗

@ $x = 4$ $f'(4)$ undefined; Change? − → 0 ✗

G has I.P @ $x = -2$ and $x = 0$ b/c $G''(x) = 0$ or undefined there and G'' changes sign.