Good afternoon: attach warm up to notes. no calculator

A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

$t(6-t)$

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

FYI: we will continue AP problems 'jigsaw' on Wednesday

A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

$x(3) - x(0)$

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

$v(t) = 6t - t^2$

$\int$

$x(t) = 3t^2 - \frac{1}{3}t^3 + C$

$x(3) = 27 - 9 + c$

$x(3) = 18 + c$

$x(0) = 0^2 + 0^3 + c = c$

$18 + c - c$

18

Area under velocity function is distance traveled (assuming positive vel.)

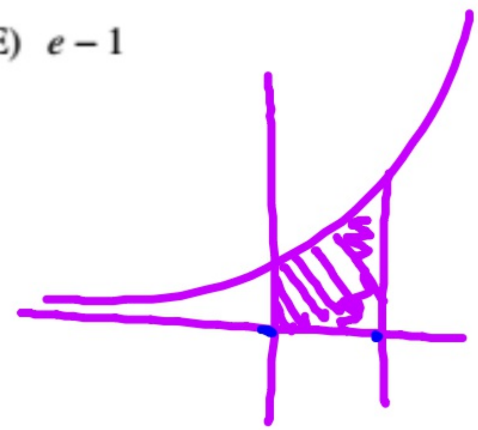
$$\int_0^3 v(t)\,dt$$

$$\int_0^3 6t - t^2\,dt$$

$$\left[3t^2 - \frac{1}{3}t^3\right]_0^3 = 18$$

What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$



$$2\int_0^2 \frac{1}{2} e^{\frac{1}{2}x}\,dx$$

$$2\left[e^{\frac{1}{2}x}\right]_0^2$$

$$2\left[e^1 - \cancel{e^0}\right]$$

$$\boxed{2e - 2}$$

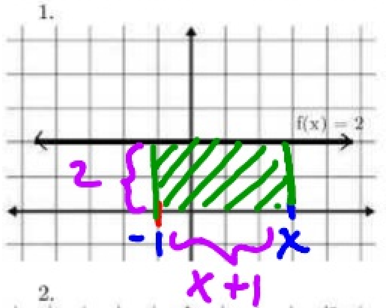
Area, Antiderivatives, Slope....guhhh???

$A(x) = \int_{-1}^{x} f(t)\,dt$

Connection between Area and Antiderivatives and Slope

For each function, use geometry to find the area A(x) *under the function* f(t) *between* -1 and some arbitrary point x (or, over the interval $[-1, x]$). *Then, find* $A'(x)$. What do you notice about $f(t)$ and $A'(x)$?

1.

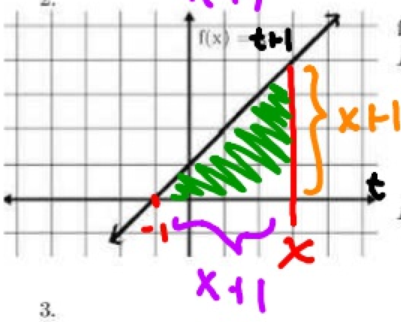


f(t) = 2

Area function A(x) = $(x+1)(2) = 2x+2$

$A'(x)$ or $\frac{dA}{dx} =$ 2

2.



f (t) = t+1

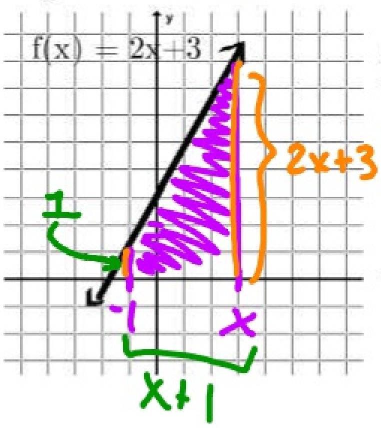
Area function A(x) = $\frac{1}{2}(x+1)(x+1)$

$\frac{1}{2}(x^2+2x+1) \rightarrow \frac{1}{2}x^2+x+\frac{1}{2}$

A'(x) or $\frac{dA}{dx} =$ $\frac{1}{2}\cdot 2x+1$

$x+1$

3.



f (t) = 2t+3

Area function A(x) = $\frac{1}{2}(2x+3+1)\cdot(x+1)$

$(x+2)(x+1)$

x^2+3x+2

A'(x) or $\frac{dA}{dx} =$

$2x+3$

Now go back and find the area under the curve using the FTC:

1. $\int_{-1}^{x} 2\,dt = \left[2t\right]_{-1}^{x} = 2x+2$

2. $\int_{-1}^{x} t+1\,dt \rightarrow \left[\frac{1}{2}t^2+t\right]_{-1}^{x} = \frac{1}{2}x^2+x-\left(\cancel{\frac{1}{2}-1}\right) = \frac{1}{2}x^2+x+\frac{1}{2}$

$-1/2$

3. $\int_{-1}^{x} 2t+3\,dt$

$\rightarrow \left[t^2+3t\right]_{-1}^{x}$

$(x^2+3x)-(1-3)$

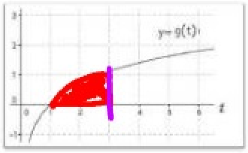
x^2+3x+2

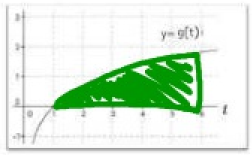
$$
\begin{tikzcd}
\text{Area under f} \arrow[rr, bend left, "\text{derivative}"] & & \text{function f} \arrow[ll, bend left, "\text{antiderivative}"]
\end{tikzcd}
$$

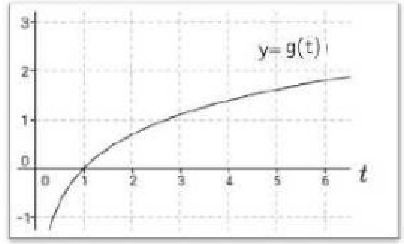
Proving the Fundamental Theorem of Calculus

A Visual and Algebraic Proof of The Fundamental Theorem of Calculus

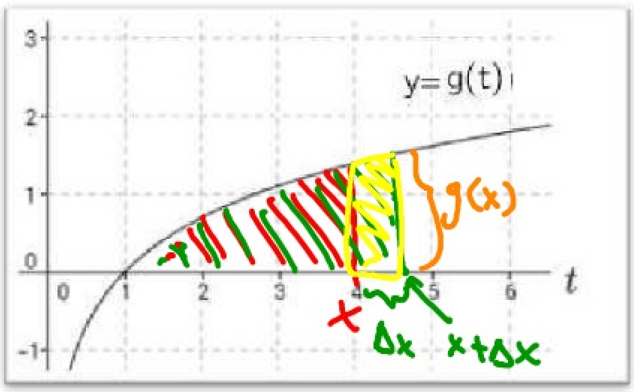
Let $f(x) = \int_a^x g(t)dt$ for the graph of g(t) shown. Let a=1 for simplicity. Sketch in $f(3)$ and $f(6)$







Let us investigate an "increment" of the area. Namely, the part added on between when you move from a value of x a value of $x + \Delta x$



Find use geometry to find the area of the incremental "rectangle"

(1) $g(x) \cdot \Delta x$

Find the area using the accumulation function.

(2) $f(x+\Delta x) - f(x)$

Since they both describe the same space, items (1) and (2) should be equal. So:

$$\frac{g(x)\cancel{\Delta x}}{\cancel{\Delta x}} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} g(x) = \boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$g(x) = f'(x)$$

$$g(x) = \frac{d}{dx}\Big[\underbrace{\int_a^x g(t)dt}_{f(x)}\Big]$$

function itself =to ⟸ "derivative of an accumulation funct"

FTC part 1

If $g(t)$ is continuous on $[a, b]$ and $f(x) = \int_a^x g(t)dt$

Then $f'(x) = g(x)$

derivatives are the inverse of definite integration

Illustration

$$\frac{d}{dx} f(x) = \cancel{\frac{d}{dx}\int_a^x} g(t)\, dt$$

$$f'(x) = g(x)$$

Some examples:

Suppose $f(x) = \int_{-\pi^2}^{x} \sin^7\left(e^{5t+2}\right) \cdot t \, dt$

$f'(x)$?

$f'(x) = \sin^7\left(e^{5x+2}\right) \cdot x$

HW
handout
#924-932

#940-952 (evens)