Welcome back and happy new year! Warm up in notes:

Suppose some unknown function has derivative
$f'(x) = 4x^3 - 2x^2 + 3x - 6$

What could have been the original function?

$$f(x) = x^4 - \frac{2}{3}x^3 + \frac{3}{2}x^2 - 6x + C$$

"constant"

Indefinite Integration aka Antidifferentiation

a quick example to dissect:

$$\int x^2 \quad dx \qquad = \quad \frac{1}{3} \; x^3 \quad + \; C$$

variable of integration (must match variable of integrand)

(usually dx, just like with derivatives)

$$\int x^2 \, dx = \frac{1}{3} x^3 + C$$

Answer

"antiderivative"

Constant of Integration

integral sign

(says, "here's a derivative... what was the original function??")

integrand

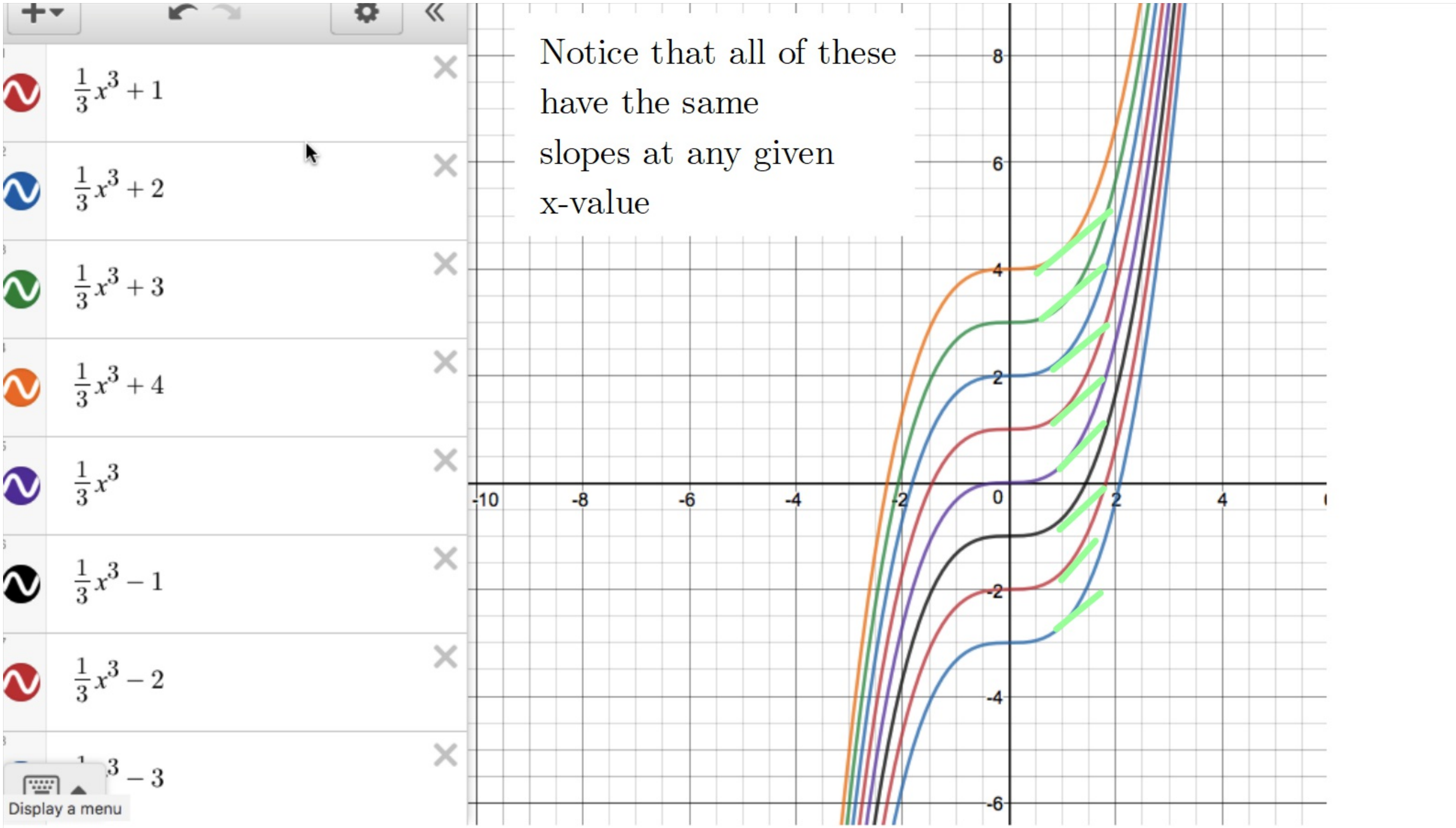
thing we are finding the antiderivative of

$$\int x^2 \, dx = \frac{1}{3} x^3 + C$$

The solution to an indefinite integral is a family of functions

x^2 is the derivative of $\frac{1}{3}x^3$, but also $\frac{1}{3}x^3 + 1$, $\frac{1}{3}x^3 + 4$, $\frac{1}{3}x^3 - 22.1$....etc

Because they all have the same slopes!! adding C is just a vertical translation



$\frac{1}{3}x^3+1$
$\frac{1}{3}x^3+2$
$\frac{1}{3}x^3+3$
$\frac{1}{3}x^3+4$
$\frac{1}{3}x^3$
$\frac{1}{3}x^3-1$
$\frac{1}{3}x^3-2$
$^3-3$
Display a menu
Notice that all of these have the same slopes at any given x-value
8
6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
2
4

How to check if you're right??

Just take the derivative of your answer and see if you get the integrand!!

integration

$$\int \cos(x) \, dx = \sin(x) + C$$

differentiation

bring your book to class today please!!!

The Reverse Power Rule

remember this? the power rule:

$$x^n \rightarrow nx^{n-1}$$

1. multiply by exponent
2. decrement exp by 1

$\frac{d}{dx}$ Power Rule

$$x^n \rightarrow nx^{n-1}$$

1. multiply by exponent
2. decrement exp by 1

$\int$ Reverse Power Rule

So in the opposite direction....

divide

1. ~~multiply~~ by exponent
2. ~~decrement~~ exp by 1

increment

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

example:

$$\int 3x^5\,dx$$

(ignore the 3, treat it as a coefficient as with derivs.)

$$3\frac{x^6}{6} + C$$

$$\frac{1}{2}x^6 + C$$

Reverse Power Rule (Add to booklets)

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

Lots to put into your booklets: p. 246 right column

Differentiation Formula	**Integration Formula**
$\frac{d}{dx}[C] = 0$	$\int 0\, dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\, dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$	$\int kf(x)\, dx = k\int f(x)\, dx$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	$\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\cancel{\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1}$ ~~Power Rule~~
$\frac{d}{dx}[\sin x] = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}[\cos x] = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}[\tan x] = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\, dx = -\csc x + C$

ADD THESE

$$\int 4x^2 - 6x^4 \quad dx$$

$$\boxed{\frac{4}{3}x^3 - \frac{6}{5}x^5 + C}$$

$$\int -\frac{2}{5}x^6 + 21\sqrt{x}\ dx$$

$$\boxed{Ax^n}$$

$$\int -\frac{2}{5}x^6 + 21x^{\frac{1}{2}}\, dx$$

$$-\frac{2}{5}\cdot\frac{x^7}{7} + \frac{21}{1}\cdot\frac{x^{3/2}}{3/2} + C$$

$$-\frac{2}{35}x^7 + \frac{\cancel{21}\,7}{1}\cdot\frac{2}{\cancel{3}}\cdot x^{3/2}$$

$$-\frac{2}{35}x^7 + 14x^{3/2} + C$$

example:

$$\int \frac{2}{\sqrt[3]{x^1}}\,dx$$

$$\sqrt[m]{x^n} = x^{\frac{n}{m}}$$

$$\frac{1}{x^n} = x^{-n}$$

$$\int \frac{2}{x^{1/3}}\,dx$$

$$\int 2x^{-\frac{1}{3}}\,d\cancel{x}$$

$$-\frac{1}{3}+1 \Rightarrow -\frac{1}{3}+\frac{3}{3} = \frac{2}{3}$$

$$2\cdot\frac{3}{2}x^{2/3}$$

$$3x^{2/3}+C \rightarrow \boxed{3\sqrt[3]{x^2}+C}$$

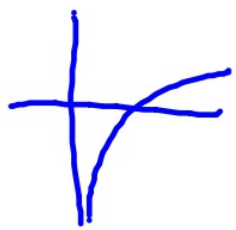
$$\int \frac{5}{x}\,dx$$

$$\cancel{\int 5x^{-1}\,dx}$$

$$\cancel{5\cdot\frac{x^0}{0}+C}$$

??

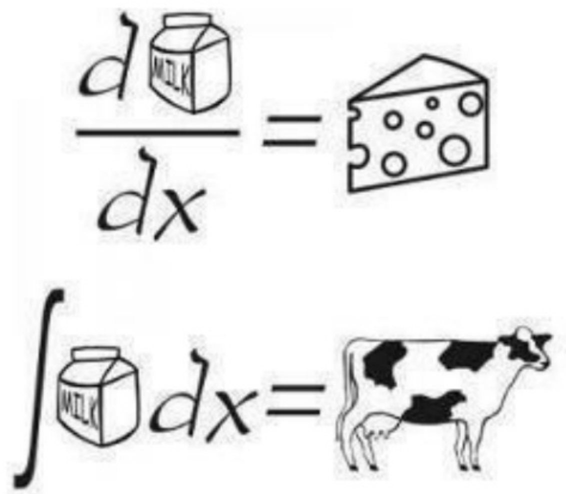
$$\underbrace{5\int \frac{1}{x}\,dx}_{5\ln|x|+C}$$



$$\int 1e^{3x}\,dx$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{1}{3}e^{3x}+C$$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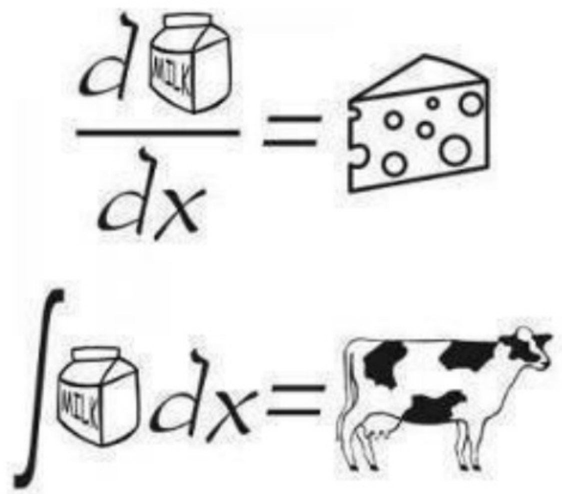
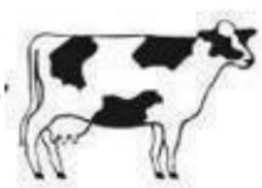
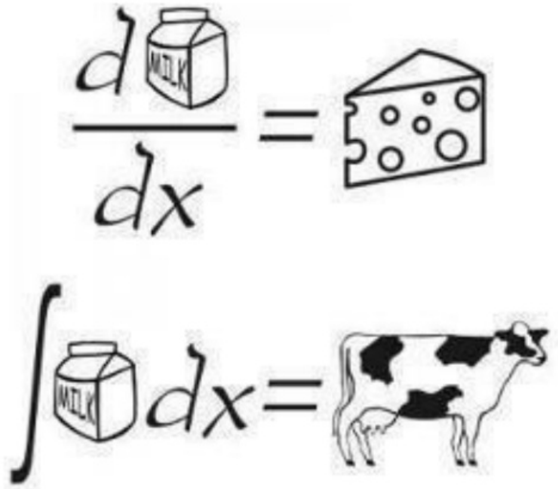
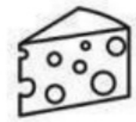


What is

$\frac{d}{dx}$  ?



What is

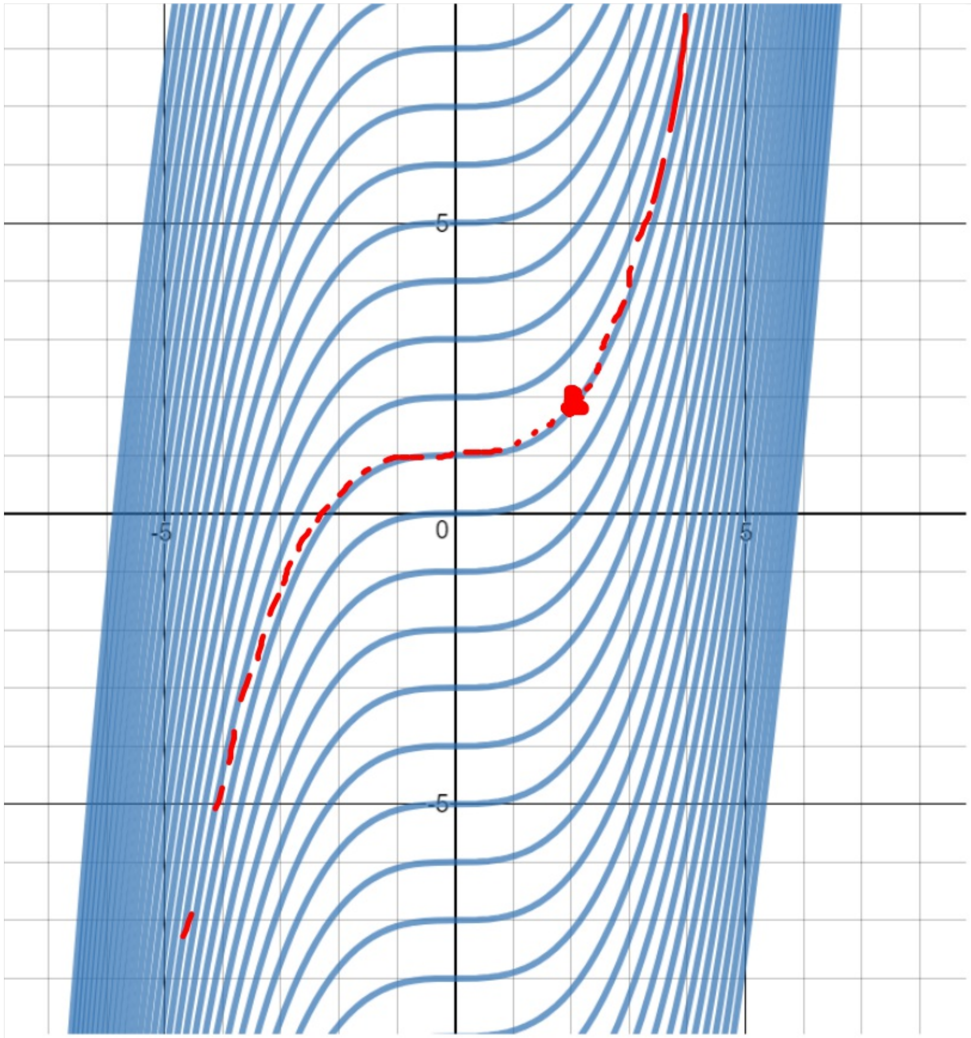
$\int$  dx ?

Finding C

Recall that the solution to

$$\int f'(x)\,dx \quad \text{is} \quad \text{a family of functions}$$

$$f(x) + C$$



Which one is it??

Knowing that a curve passes through a particular point allows us to move from a ***general solution*** to a ***particular solution***

Suppose $\frac{dy}{dx} = \frac{1}{4}x + 2$ and $y(-4)=5$ Find the particular solution, y

$$\cancel{dx}\left(\frac{dy}{\cancel{dx}} = \frac{1}{4}x + 2\right)dx$$

multiply both sides by dx

$$\int 1 \cdot dy = \int \frac{1}{4}x + 2 \; dx$$

integrate both sides

$$1y + \cancel{C} = \frac{1}{4} \cdot \frac{x^2}{2} + 2x$$

$$y = \frac{1}{8}x^2 + 2x + C$$

general solution

$$5 = \frac{1}{8}(-4)^2 + 2(-4) + C$$

$$5 = 2 - 8 + C$$

$$5 = -6 + C$$

$$C = 11$$

$$y = \frac{1}{8}x^2 + 2x + 11$$

classwork/homework
- p 251: #7-26, 50

Don't forget calcchat
or just take the derivative to check :)