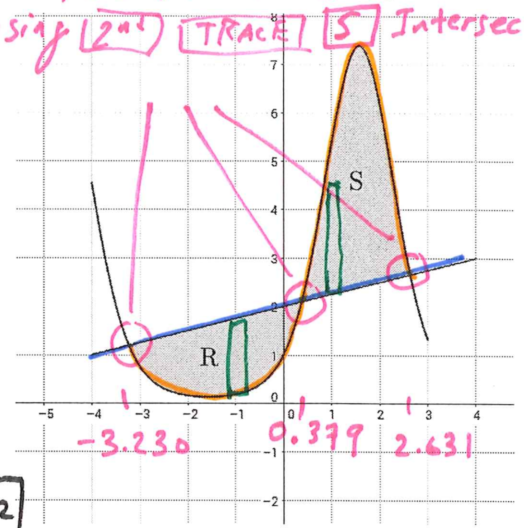
I-A4b

Big Ol Practice Assessment

1. Let $f(x) = e^{2\sin x}$ and $g(x) = \frac{1}{4}x + 2$ be the boundaries of the regions R and S. Find the total area of R and S.

Find these in Calc. using $\boxed{2^{nd}}$ $\boxed{TRACE}$ $\boxed{5}$ Intersect



$$R = \int_{-3.230}^{0.379} \frac{1}{4}x + 2 - \left(e^{2\sin x}\right) dx \xrightarrow{\boxed{MATH}\ +\boxed{9}} 4.196$$

$$S = \int_{0.379}^{2.631} e^{2\sin x} - \left(\frac{1}{4}x + 2\right) dx \xrightarrow{\boxed{MATH}\ +\boxed{9}} 6.457$$

Total: $\boxed{10.653\, u^2}$

I-U7: Given $\int_0^5 f(x)\,dx = 10$ $\quad \int_5^7 f(x)\,dx = 3$ $\quad \int_{-2}^5 f(x)dx = -2$ $\quad$ Find each of the following:

2. $\int_7^{-2} f(x)\,dx$

$\to -\int_{-2}^{7} f(x)\,dx \to -\left[\int_{-2}^{5} f(x)\,dx + \int_5^7 f(x)\,dx\right]$

$-\left[-2 + 3\right] \to -1$

3. $\int_0^{-2} f(x)dx$

$= -\int_{-2}^{0} f(x)\,dx \Rightarrow -\left[\int_{-2}^{5} f(x)dx + \int_5^0 f(x)\,dx\right] \to -\left[\int_{-2}^{5} f(x)dx - \int_0^5 f(x)dx\right]$

$-\left[-2 - 10\right]$

12

I-U4

Let $f(x) = \int_{-4}^{x^2} 4t^2 - 4t + 1\, dt$.

4. Find $f'(x)$. Simplify your answer.

$f'(x) = \left(4(x^2)^2 - 4x^2 + 1\right)(2x)$

$f'(x)\ \left(4x^4 - 4x^2 + 1\right)(2x) \to 8x^5 - 8x^3 + 2x$

5. Find all intervals where $f(x)$ is decreasing. Justify your answer.

$f'(x) < 0$

$f'(x) = (4x^4 - 4x^2 + 1)(2x)$

$(2x^2 - 1)(2x^2 - 1)(2x) = 0$ Find C.N.

$2x^2 - 1 = 0$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{1/2}$

$\approx \pm 0.707$

$x = 0$

← C.N.

f': $-$ $\quad -$ $\quad +$ $\quad +$

x: $-\sqrt{1/2}$ $\quad$ $-.5$ $\quad$ 0 $\quad$ $.5$ $\quad$ $\sqrt{1/2}$

Plug in test values into f'

$f'(1000) = (+)(+)(+) = +$

$f'(-1000) = (+)(+)(-) = -$

$f'(0.5) = (-)(-)(+) = +$

$f'(-0.5) = (-)(-)(-) = -$

f dec. on $(-\infty, -\sqrt{1/2})$ $(-\sqrt{1/2}, 0)$ b/c $f' < 0$

I-U9

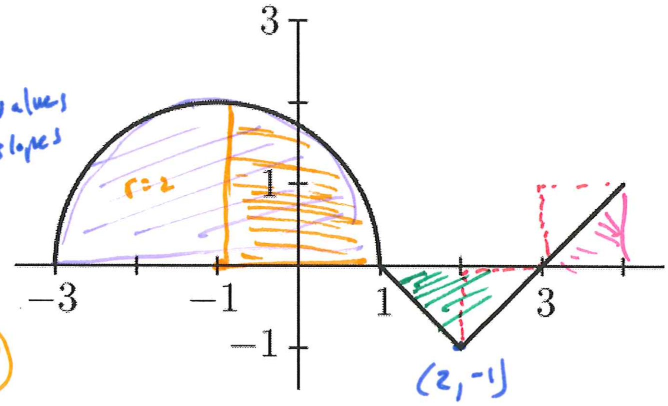
The function $a(t)$ is shown over [-3,4] and consists of line segments and a semicircle.

Let $Q(x) = \int_1^x a(t)\,dt$ $\xrightarrow[\text{Part 1}]{\text{FTC}}$

$Q = \int a(t)$

$Q' = a(x)$ ← values

$Q'' = a'(x)$ ← slopes



6. Find $Q(-1)$, $Q'(2)$, and $Q''(3)$.

$Q(-1) = \int_1^{-1} a(t)\,dt$

$-\int_{-1}^{1} a(t)\,dt = -\frac{1}{4}(\pi)(2^2) = -\pi$

$Q'(2) = a(2) = \boxed{-1}$

$Q''(3) = a'(3) = \boxed{1}$

Slope of a @ x = 3

7. Find the relative minima of $Q(x)$, if any, over [-3,3]. Justify your answer.

$Q'(x)$ changes sign $- \to +$

↳ AKA, $a(x)$. Only such sign change @ $\boxed{x = 3}$

b/c $Q'(x)$ "($a(x)$)" changes sign $- \to +$ and $Q'(3) = 0$.

8. Find where $Q(x)$ has an absolute minimum value on $[-3,3]$. Show all calculations.

Possible only @ Relative ~~minima~~ minima and endpoints.

Rel min @ x = 3 $\quad Q(3) = \int_1^3 a(t)\,dt = \underbrace{\tfrac{1}{2}(2)}_{\text{base}}\underbrace{(-1)}_{\text{height}} = -1$

Endpts

$x = -3 \quad Q(-3) = \int_1^{-3} a(t)\,dt \Rightarrow -\int_{-3}^{1} a(t)\,dt = -\frac{1}{2}\cdot\pi(2^2)$

$= -2\pi$

Smallest

$x = 4 \quad Q(4) = \int_1^4 a(t)\,dt = -1 + \frac{1}{2} = -1/2$

$Q(x)$ has abs min @ $x = -3$.

I-A7b

$F(10) = 8$ (10 am, 8 mb used)

9. It's 10am and Frank has already used 8 mb of data on his cell phone. From 10am to midnight (t=24), his data usage rate can be modeled by the differentiable function $f(t) = \sin\left(\frac{\pi}{8}t\right) + 1$ mb/hr. First, write an equation that includes an integral that will give the amount of data Frank has used as of midnight. Then, find that amount and include units in your answer.

Net change

$$F(t) = F(10) + \int_{10}^{t} f(x)\,dx$$

$$\boxed{F(t) = 8 + \underbrace{\int_{10}^{t} \sin\left(\tfrac{\pi}{8}x\right)+1\,dx}_{\text{accumulated data usage from 10 am onwards.}}}$$

hrs after midnight

data used as of 10 am

$$F(24) = 8 + \int_{10}^{24} \sin\left(\tfrac{\pi}{8}x\right)+1\,dx$$

$$= 8 + 14.746$$

$$\boxed{= 22.746 \text{ mb}}$$

I-A7a

10. Find the average value of $f(x) = \frac{1}{x}$ over the interval [1,3]

a b

$$\frac{1}{b-a}\int_a^b f(x)\,dx \rightarrow \frac{1}{3-1}\int_1^3 \frac{1}{x}\,dx$$

$$\frac{1}{2}\left[\ln|x|\right]_1^3$$

$$\underbrace{\frac{1}{2}\left[\ln 3 - \ln 1\right]}_{\text{okay Answer}} \rightarrow \frac{1}{2}\ln\frac{3}{1} \rightarrow \ln 3^{1/2}$$

$$\boxed{\ln\sqrt{3}}$$

great answer!

11. Let $Z'(t) = 1 - \cos(\frac{\pi t}{5})$ model the rate, in hundreds of people per hour, enter an amusement park. Using correct units, explain the meaning of $\frac{1}{5}\int_2^7 Z'(t)\,dt$ in context. Then, find its value.

Average Value of $Z'(t)$, which is a rate in hundreds of ppl/hr.

$\frac{1}{5}\int_2^7 Z'(t)\,dt$ is the Average Rate, in 100's of ppl per hour, of people entering the amusement park from $t=2$ to $t=7$.

$$\frac{1}{5}\int_2^7 1-\cos\left(\frac{\pi t}{5}\right)dt \approx \frac{1}{5}(8.027) \approx 1.605 \approx$$

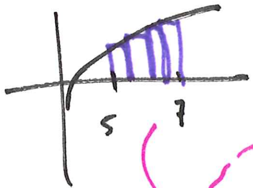
$$\boxed{\approx 160 \text{ ppl/hr}}$$

I-U3a

12. Find the left rectangle approximation for $\int_5^7 \overbrace{\ln(3x)}^{f(x)}\,dx$ using $\overset{n}{4}$ rectangles of equal width. [3 decimal places of accuracy]. Is the approximation an under or an overestimate? Justify your response.

$\Delta x = \frac{b-a}{n} = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

x	5	5.5	6	6.5
$f(x)$	2.708	2.803	2.890	2.970

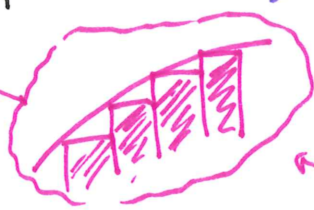


$$\int_5^7 \ln(3x)\,dx \approx \underbrace{\tfrac{1}{2}}_{\Delta x}\underbrace{(2.708 + 2.803 + 2.890 + 2.970)}_{\sum f(x_i)}$$

$$\approx \tfrac{1}{2}(11.371)$$

$$\boxed{\approx 5.686}$$

under Approx. b/c f is increasing.



I-U3c

13. An awesome rocket ship is in the air and doing cool rocket things. Its velocity $v(t)$ is a differentiable, strictly increasing function. Selected values are given below. Using correct units, explain the meaning of $\int_2^{10} v(t)\,dt$ in the context of this problem. Then, approximate the value of $\int_2^{10} v(t)\,dt$ using the 4 trapezoids indicated by the table.

Distance traveled by rocket, in meters, from $t = 2$ sec to $t = 10$ sec.

$\Delta x = 2$

t	2	4	6	8	10
$v(t)$, m/s	12	18	27	38	52

$$\frac{2}{2}\left(12 + 2(18 + 27 + 38) + 52\right) = 1(230) = \boxed{230 \text{ meters}}$$

I-A1

14. $\int \frac{\sqrt{x^3}-2}{\sqrt{x}}\,dx = \int \frac{x^{3/2} - 2}{x^{1/2}}\,dx \rightarrow \int \frac{x^{3/2}}{x^{1/2}} - \frac{2}{x^{1/2}}\,dx$

$\int x^1 - 2x^{-1/2}\,dx$ rev. pwr rule

$\frac{1}{2}x^2 - 2 \cdot 2x^{1/2} + C$

$\boxed{\frac{1}{2}x^2 - 4x^{1/2} + C}$ #14

15. $\frac{1}{2}\int \frac{2}{x}\,dx \Rightarrow \underbrace{\frac{1}{2}\cdot 2}\int \frac{1}{x}\,dx$

$1 \cdot \ln|x| + C$

$= \boxed{\ln|x| + C}$ #15

16. $\int \csc^2\theta\,d\theta$

Recall: $\frac{d}{dx}\cot x = -\csc^2 x$

$\boxed{-\cot(\theta) + C}$ #16

17. $\int 4^t \ln 4\,dt$

Recall: $\frac{d}{dx}a^x = a^x \cdot \ln a$

$\boxed{4^t + C}$ #17