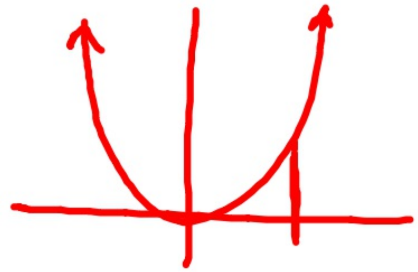
Using the FTC

Suppose $f(x) = \int_x^3 t^2\,dt$



Write the equation of the line tangent to f(x) where x=3.

$f(x) = -\int_3^x t^2\,dt$

$\cdot\ f(3) = -\int_3^3 t^2\,dt = 0$

$(3, 0)$

$y - y_1 = m(x - x_1)$

$y - 0 = m(x - 3)$

$\boxed{y = -9(x-3)}$

$\cdot\ f'(3)\ ?$

$f'(x) = \frac{d}{dx}\left[-\int_3^x t^2\,dt\right]$

$-\cancel{\frac{d}{dx}\left[\int_3^x\right.} t^2\,dt\Big]$

$f'(x) = -x^2$

$f'(3) = -9$

PEMDAS

$-3 \div 3 \cdot 1$

$4 - 3 + 5$

Function Types You've Studied

1. constant/linear
2. absolute value
3. quadratic
4. cubic
5. quartic
6. polynomial
7. rational $\longrightarrow \dfrac{3}{x-4}$
8. exponential
9. logarithmic
10. trigonometric
11. inverse trigonometric
12. Accumulation func.

$$\frac{g(x)\cancel{\Delta x}}{\cancel{\Delta x}} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} g(x) = \boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

$$g(x) = f'(x)$$

$$g(x) = \frac{d}{dx}\left[\int_a^x g(t)\,dt\right]$$

$$\frac{d}{dx}\left[\int_a^x g(t)\,dt\right] = g(x)$$

By the result on the bottom of pg1: <u>derivative is the inverse of definite integration.</u> $\int x^2 dx$

By common sense (construction) the <u>derivative is the inverse of the antiderivative.</u>

THEREFORE:

Definite Integration = antiderivatives

$\int_3^5 x^2 dx$ $\qquad$ $\int x^2 dx$

FTC Part 2

Now, why is this true: $\int_a^b g(x)dx = f(b) - f(a)$ where $f'(x) = g(x)$

Let $f(x) = \int_a^x g(t)\,dt$

I: $f(a) = \int_a^a g(t)\,dt = 0$

II: $f(b) = \int_a^b g(t)\,dt$

[redacted]

I: $f(a) = \int_a g(t)\,dt$

II: $f(b) = \int_a^b g(t)\,dt$

$$\int_a^b g(t)\,dt = \int_a^a g(t)\,dt + \int_a^b g(t)\,dt$$

$$\int_a^b g(t)\,dt = \underbrace{-\int_a^a g(t)\,dt}_{\mathrm{I}} + \underbrace{\int_a^b g(t)\,dt}_{\mathrm{II}}$$

$$\int_a^b g(t)\,dt = -f(a) + f(b)$$

$$\boxed{\int_a^b g(t)\,dt = f(b) - f(a)} \;\blacksquare$$

Q.E.D.

- Using FTC 2: evaluating definite integrals
- Using FTC 1: taking derivatives of accumulation functions

ex $\int_2^5 x^3\,dx = \left[\frac{1}{4}x^4\right]_2^5 = \frac{1}{4}(5^4) - \cancel{\frac{1}{4}(32)}^{\,8}$

$$\boxed{\frac{625}{4} - 8}$$

ex If $y = \int_7^x \sin^3(t^2)\,dt$

What's y'?

$$\boxed{y' = \sin^3(x^2)}$$

How is this question different?

Find $f'(x)$

If $f(x) = \int_{3}^{5x^2-2} \cos(t)\,dt$

$f'(x) = \cos(5x^2-2) \cdot 10x$

Chain Rule version of FTC 1

$$\frac{d}{dx}\int_a^{f(x)} g(t)\,dt = g(f(x)) \cdot f'(x)$$

EX

Find g' where $g(x) = \int_{x^3}^{3} \sqrt{t^3}\, dt$

$\frac{d}{dx} g(x) = \cancel{\frac{d}{dx}} - \int_{3}^{x^3} \sqrt{t^3}\, dt$

$g'(x) = -\left[\sqrt{(x^3)^3} \cdot 3x^2\right]$

$= -\left[x^{9/2} \cdot 3x^2\right]$

$\boxed{= -3x^{13/2}}$

$(a^n)^m = a^{n \cdot m}$

$a^n \cdot a^m = a^{n+m}$

How can we use the fundamental theorem of calculus?

Let $f(x) = \int_{2}^{x} t^2+3t-10\ dt$

Where is f increasing? Justify.

$f'(x) = x^2+3x-10 = 0$

$(x+5)(x-2) = 0$

$\left.\begin{array}{l} x=-5 \\ x=2 \end{array}\right]$ $\underline{\underline{C.N.}}$

$f' > 0$?

$+ \quad - \quad +$

x: -5, 2

$\boxed{(-\infty,-5),(2,\infty)}$

Where does f have inflection points? Justify.

Sign change in f''

$f'' = 2x+3 = 0$

$\underline{x=-1.5}$

terrace pt.

$- \quad +$

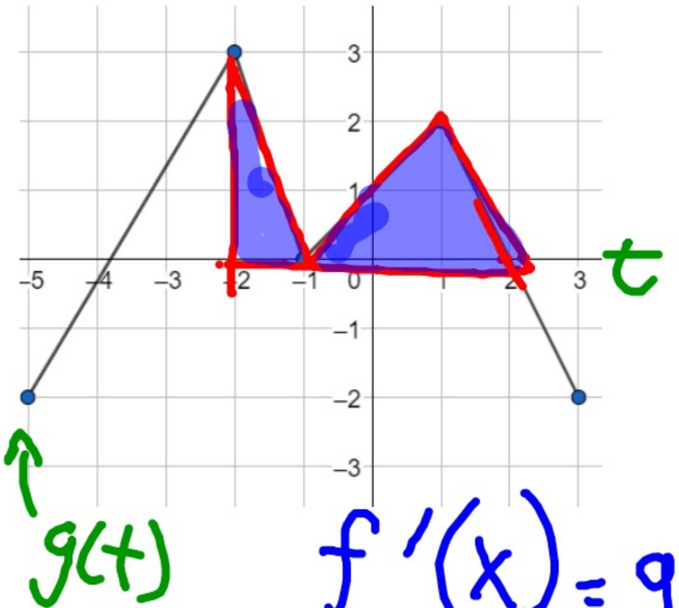
-1.5

f has I.P. @ $x=-1.5$ b/c f'' changes signs.

Better understanding accumulation functions

Let F be defined such that $F(x) = \int_{-2}^{x} g(t)dt$

$F(2) = \int_{-2}^{2} g(t)dt = 4.5$



F(2)?

F'(2)? $g(2) = 0$

F''(2)? $g'(2) = -\frac{2}{1}$ ← slope @ x=2

Where is F increasing and concave down?

$f'(x) = g(x)$

$f''(x) = g'(x)$

HW
handout